

## Section 1: The scalar product

### Exercise level 3

1. Given that  $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j}$  is the position vector of a point  $A$ ,  $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j}$  is the position vector of a point  $B$  and  $\theta$  the angle between  $\mathbf{a}$  and  $\mathbf{b}$ , use the cosine rule in the triangle  $OAB$  to show that  $|\mathbf{a}||\mathbf{b}|\cos\theta = a_1b_1 + a_2b_2$ .
2. Use the scalar product to prove that
$$(ac + bd)^2 \leq (a^2 + b^2)(c^2 + d^2)$$
for all real  $a, b, c$  and  $d$ . Find a condition on  $a, b, c$  and  $d$  for the equality to hold.
3. Show that if  $\mathbf{u} + \mathbf{v}$  and  $\mathbf{u} - \mathbf{v}$  are perpendicular then  $\mathbf{u}$  and  $\mathbf{v}$  must have the same modulus.