Section 2: Proof by induction

Exercise level 2

In Questions 1 to 12 prove the given result by induction.

1. $\sum_{r=1}^{n} r^2(r+1) = \frac{n(n+1)(n+2)(3n+1)}{12}$

2. $\sum_{r=1}^{n} \frac{r}{2^r} = 2 - \frac{(n+2)}{2^n}$

3. $\sum_{r=1}^{n} 2\times 3^r = 3\left(3^n - 1\right)$

4. The sum of the first n terms of the series $(1 \times 3) + (2 \times 4) + (3 \times 5) + \ldots\ldots\ldots..$ is $\frac{n(n+1)(2n+7)}{6}$

5. The sum of the first n terms of the series $\frac{1}{1\times 3} + \frac{1}{3\times 5} + \frac{1}{5\times 7} + \ldots.. + \frac{1}{(2r-1)(2r+1)}$ is $\frac{n}{(2n+1)}$

6. $\sum_{r=1}^{n} 2^{r-1} = 2^n - 1$

7. For a sequence defined by $u_{n+1} = 3u_n + 2$ and $u_1 = 1$ for $n \geq 1$, $u_n = 2\left(3^{n-1}\right) - 1$.

8. Given that $u_{n+1} = 2u_n + 1$ where n is a positive integer and $u_1 = 5$, $u_n = 3\times 2^n - 1$.

9. If $\mathbf{A} = \begin{pmatrix} 3 & -1 \\ 4 & -1 \end{pmatrix}$, $\mathbf{A}^n = \begin{pmatrix} 2n+1 & -n \\ 4n & 1-2n \end{pmatrix}$ where n is a positive integer.

10. If $\mathbf{M} = \begin{pmatrix} 2 & 6 \\ 1 & 3 \end{pmatrix}$, $\mathbf{M}^n = 5^{n-1}\begin{pmatrix} 2 & 6 \\ 1 & 3 \end{pmatrix}$ where $n \geq 1$.

11. $n^3 + 3n^2 - 10n$ is a multiple of 3 for all positive integers n.

12. $3^{2n} - 1$ is a multiple of 8 for all positive integers n.