Section 2: Proof by induction

Exercise level 1

1. In this question you will prove that for the sequence $u_1 = 2$, $u_{n+1} = 2u_n - 3$, the nth term of the sequence is given by $u_n = 3 - 2^{n-1}$.
   (i) Show that the result is true for $n = 1$.
   (ii) Assume that $u_k = 3 - 2^{k-1}$. Use this and the recurrence relation $u_{k+1} = 2u_k - 3$ to write down an expression for u_{k+1}.
   (iii) Show that your answer to (ii) can be written in the form $u_{k+1} = 3 - 2^k$.
   (iv) Write the conclusion to your proof.

2. In this question you will prove that for the matrix $\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 2 & 1 \end{pmatrix}$, $\mathbf{A}^n = \begin{pmatrix} 2^n & 0 \\ 2^{n+1} - 2 & 1 \end{pmatrix}$ for all $n \geq 1$.
   (i) Show that the result is true for $n = 1$.
   (ii) Assume that $\mathbf{A}^k = \begin{pmatrix} 2^k & 0 \\ 2^{k+1} - 2 & 1 \end{pmatrix}$. Multiply this by **A** to find the matrix $\mathbf{A}^{k+1}$.
   (iii) Show that your answer to (ii) can be written in the form
   $\mathbf{A}^{k+1} = \begin{pmatrix} 2^{k+1} & 0 \\ 2^{k+2} - 2 & 1 \end{pmatrix}$.
   (iv) Write the conclusion to your proof.

3. In this question you will prove that the sum of the series $1 + 4 + 7 + ... + (3n - 2)$ is $\frac{1}{2}n(3n-1)$.
   (i) Show that the result is true for $n = 1$.
   (ii) Write down the (k+1)th term of the series.
   (iii) Assume that the sum of the first k terms is $\frac{1}{2}k(3k-1)$
   So the sum of the first (k+1) terms is $\frac{1}{2}k(3k-1)$ + the (k+1)th term.
   Use this, and your answer to (ii), to write down an expression for the sum of the first (k+1) terms.
   (iv) Show that your answer to (iii) can be written in the form $\frac{1}{2}(k+1)(3(k+1)-1)$.
   (v) Write the conclusion for your proof.

4. Follow the same method as for question 3 to prove that $\sum_{r=1}^{n}(2r-3) = n(n-2)$.

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