Section 2: Complex roots of polynomials

Exercise level 2

1. An equation is of the form f(z) = 0, where f(z) is a polynomial of degree 4 with real coefficients.
   Two of the roots of the equation are (3 + i) and (1 + 3i).
   Find the equation.

2. Given that 1 + i is a root of the equation $z^3 - 2z + k = 0$ find the other roots and a value for *k*.

3. Show that $z = -1 + \mathrm{i}$ is a root of the equation $z^4 - 2z^3 - z^2 + 2z + 10 = 0$ and find the remaining roots.

4. One root of the equation $z^4 - 6z^3 + 18z^2 - 30z + 25 = 0$ is $1 + 2\mathrm{i}$.
   Solve the equation.

5. Given that one root of the equation $z^3 + az + b = 0$ is $p + q\mathrm{i}$ where *a* and *b* are real and *b* is not zero, prove that
   (i) $2p(p^2 + q^2) = b$
   (ii) $-3p^2 + q^2 = a$
   (iii) *p* is a root of the equation $8x^3 + 2ax - b = 0$.