Section 1: Roots and coefficients

Exercise level 2

1. One root of $2x^2 - kx + k = 0$ is twice the other. Find k. You may assume that $k \neq 0$.

2. The two roots of $x^2 + (7-p)x - p = 0$ differ by 5. Find the possible values for p.

3. If α and β are the roots of $ax^2 + bx + c = 0$ prove that
   (i) if $\beta = 4\alpha$ then $4b^2 = 25ac$
   (ii) if $\beta = \alpha + 1$ then $a^2 = b^2 - 4ac$

4. If the roots of $x^3 + 5x^2 + hx + k = 0$ are α, 2α, and $\alpha + 3$ find α, h and k.

5. Solve the equation $24x^3 + 28x^2 - 14x - 3 = 0$ given that the roots are of the form α, $\frac{\alpha}{r}$ and αr .

6. The equation $6x^3 + 11x^2 + kx - 9 = 0$ has roots α, $\frac{1}{\alpha}$ and β.
   Find the value of k and solve the equation.

7. The roots of $x^3 - 2x^2 - x + 2 = 0$ are α, β, and γ. Find equations which have roots
   (i) $2\alpha, 2\beta, 2\gamma$
   (ii) $\alpha - 3, \beta - 3, \gamma - 3$

8. The roots of $x^4 + ax^3 + bx^2 + cx + d = 0$ are α, β, γ, and δ. Given that $\alpha + \beta = \gamma + \delta$, show that $a^3 + 8c = 4ab$.