Summary sheet: Vectors

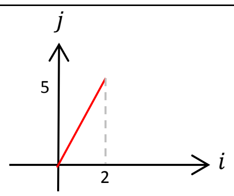
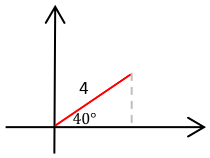
J1 Use vectors in two dimensions
J2 Calculate the magnitude and direction of a vector and convert between component form and magnitude/direction form
J3 Add vectors diagrammatically and perform the algebraic operations of vector addition and multiplication by scalars, and understand their geometrical interpretations
J4 Understand and use position vectors; calculate the distance between two points represented by position vectors
J5 Use vectors to solve problems in pure mathematics and in context, including forces

Vectors

A vector has both magnitude (size) and direction. To denote that something is a vector it can be written in bold (e.g. **u**), underlined (e.g. $\underline{u}$) or with an arrow above (e.g. $\vec{u}$). Remember that $|\vec{a}|$ means the magnitude (size) of a.

Vector forms

There are 2 different ways of writing a vector: **component form** and **magnitude/direction form**. Both forms tell you how to draw the vector but component form gives you the co-ordinates (along and up) and magnitude/direction tells you the angle to turn through and the length of the line.

Form:	Uses:	e.g.	Meaning:	
Component:	$\mathbf{i}$ and $\mathbf{j}$	$2\mathbf{i} + 5\mathbf{j}$	Go along 2 and up 5	
Magnitude/direction:	r and θ	$(4, 40°)$	Turn 40° from the horizontal and draw a line with length of 4	

Converting between component form and magnitude/direction form

If you find it difficult to remember how to convert between the forms, try sketching the graph to remind you. You can see from the graphs above, that, if you have the $\mathbf{i}$ and $\mathbf{j}$ values you should use Pythagoras to find r and tan to find θ. If you have r and θ then you would use *cos* to find the $\mathbf{i}$ value and *sin* to find the $\mathbf{j}$ value.

Component ⟷	magnitude/direction
Need to find $x\mathbf{i}+y\mathbf{j}$	**Need to find r and θ**
$x = r\cos\theta$	$r = \sqrt{x^2 + y^2}$
$y = r\sin\theta$	$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

Remember when finding θ that your calculator will always give values between $0°$ and $90°$ or between $0°$ and $-90°$. This will be incorrect if your vector is in a different quadrant.

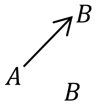
So either: enter **<u>positive</u>** $\mathbf{i}$ and $\mathbf{j}$ values in your calculator then do a **sketch** to see where the angle is
or: put the correct signs in your calculator and remember that if $\boldsymbol{i} < \mathbf{0}$ (i.e. negative) **add $\mathbf{180°}$** to your answer.

Summary sheet: Vectors

Adding and multiplying vectors

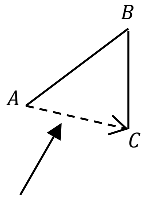
To add vectors **diagrammatically** you would draw them as a chain with the 2nd vector starting where the 1st one ended. The shortest route from start to finish is then drawn on and is called the resultant vector.

e.g. if $\overrightarrow{AB}$ is the vector



and $\overrightarrow{BC}$ is the vector

Drawing them as a chain gives:



The resultant vector
(shortest route from start to finish)

Therefore, adding the vectors gives: $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

To add vectors **algebraically** you add the $\hat{\mathbf{i}}'s$ and add the $\hat{\mathbf{j}}'s$:

e.g. add the vectors: $\underline{a} = 5\mathbf{i} + 3\mathbf{j}$ and $\underline{b} = \mathbf{i} - 8\mathbf{j}$

$$\underline{a} + \underline{b} = 6\mathbf{i} - 5\mathbf{j}$$

To **multiply by a scalar**, multiply each component.

e.g. if $\underline{a} = 5\mathbf{i} + 3\mathbf{j}$ find $3\underline{a}$

$$3\underline{a} = 3(5\mathbf{i} + 3\mathbf{j}\,) = 15\mathbf{i} + 9\mathbf{j}$$

When multiplying by a scalar remember:

If you multiply by a +ve number (e.g. n): direction stays the same, size becomes n times bigger.

If you multiply by a -ve number (e.g. -n): direction reverses, size becomes n times bigger.

Position Vectors

A position vector starts at the origin.

e.g. Point A $(-3,\ 2)$ has position vector $\overrightarrow{OA} = -3\boldsymbol{i} + 2\boldsymbol{j}$

Some things to remember:

$\overrightarrow{\boldsymbol{AO}} = -\overrightarrow{\boldsymbol{OA}}$ — Going from A to O is the same as going from O to A in reverse.

$\overrightarrow{\boldsymbol{AB}} = \overrightarrow{\boldsymbol{OB}} - \overrightarrow{\boldsymbol{OA}}$ — Easy way to remember this: think $\overrightarrow{AB} = \boldsymbol{b} - \boldsymbol{a}$, then change them into position vectors.

Midpoint $\overrightarrow{\boldsymbol{OM}} = \overrightarrow{\boldsymbol{OA}} + \frac{\mathbf{1}}{\mathbf{2}}\overrightarrow{\boldsymbol{AB}}$ — To get from the origin to the midpoint, start at A and add half of the distance between A and B.

Summary sheet: Vectors

Finding the distance between 2 points

When finding the distance between 2 points you are finding the length of the line between the points, and so you would find the horizontal and vertical distance between each point and then use Pythagoras.

$$\boxed{|\overrightarrow{\boldsymbol{AB}}| = \sqrt{(\boldsymbol{i_2} - \boldsymbol{i_1})^2 + (\boldsymbol{j_2} - \boldsymbol{j_1})^2}}$$

e.g. Find the distance between the points $\boldsymbol{A}$ and $\boldsymbol{B}$ given the vectors $|\overrightarrow{OA}| = 3\boldsymbol{i} - \boldsymbol{j}$ and $|\overrightarrow{OB}| = 5\boldsymbol{i} + 2\boldsymbol{j}$

$$|\overrightarrow{AB}| = \sqrt{(i_2 - i_1)^2 + (j_2 - j_1)^2}$$

$$|\overrightarrow{AB}| = \sqrt{(5 - 3)^2 + (2 - -1)^2}$$

$$|\overrightarrow{AB}| = \sqrt{(2)^2 + (3)^2}$$

$$|\overrightarrow{AB}| = \sqrt{13}$$

