Summary sheet: Integration

H1 Know and use the Fundamental Theorem of Calculus
H2 Integrate x^n (excluding n = -1), and related sums, differences and constant multiples
H3 Evaluate definite integrals; use a definite integral to find the area under a curve

Notation

Integration questions are usually given as follows:

$$\int (3x^2 + 7x - 1)\,\mathrm{d}x$$

Means: *Integrate the following* (the $\int$ sign) *with respect to x* (the $\mathrm{d}x$)

The fundamental theorem of calculus

Remember that integration is the reverse of differentiation (they 'undo' each other). The rule for differentiation is: expression X power, then reduce the power by 1. So integration is the opposite:

$$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1} + c$$

and $$\int kx^n\,\mathrm{d}x = \frac{kx^{n+1}}{n+1} + c$$

Add 1 to the power then divide the expression by the <u>new</u> power

Remember for each term: **power +1** then **divide by new power**.

e.g. $$\int (15x^2 - 6x + 7)\,\mathrm{d}x$$

$$= \frac{15x^3}{3} - \frac{6x^2}{2} + 7x + c$$

Remember to include the c because there could have been a number in the original that disappeared when differentiating.

$$= 5x^3 - 3x^2 + 7x + c$$

You will only be able to find c if you are given some more information.

e.g. for the above example, when $x = 1$, $y = 6$. Find the value of c

You have found that $y = 5x^3 - 3x^2 + 7x + c$, so just substitute the given values in to find c.

$$6 = 5(1)^3 - 3(1)^2 + 7(1) + c$$
$$6 - 5 + 3 - 7 = c$$
$$c = -3$$

So the final answer is: $y = 5x^3 - 3x^2 + 7x - 3$

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Definite integrals

A definite integral has limits. To evaluate a definite integral you integrate as normal then substitute the top limit and the bottom limit and subtract.

$$[\boldsymbol{top\ limit}] - [\boldsymbol{bottom\ limit}]$$

Remember that definite integration is used to find the area under a curve ("Under the curve" means between the curve and the x-axis).

e.g. Find the area enclosed by the curve $y = -x^2 + 7x - 10$ and the lines $x = 3$ and $x = 5$

Set up the integration:

Upper limit → 5, *Lower limit* → 3

$$\int_3^5 (-x^2 + 7x - 10)\mathrm{d}x$$

Integrate:

$$= \left[-\frac{x^3}{3} + \frac{7x^2}{2} - 10x\right]_3^5$$

Notice that you don't need to include the c, because you are going to subtract, so it would cancel out anyway.

$[top\ limit] - [bottom\ limit]$

$$= \left[-\frac{5^3}{3} + \frac{7(5)^2}{2} - 10(5)\right] - \left[-\frac{3^3}{3} + \frac{7(3)^2}{2} - 10(3)\right]$$

$$= \left[-\frac{25}{6}\right] - \left[-\frac{15}{2}\right]$$

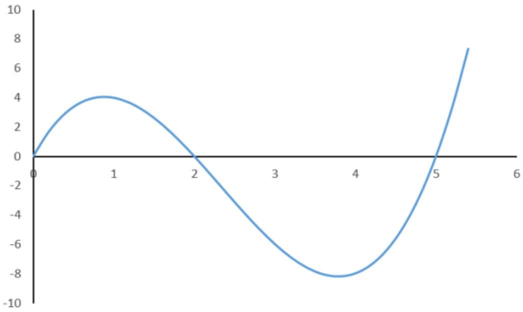
$$= \frac{10}{3}\ (3.3)$$

You have found the area under the curve, between $x = 3$ and $x = 5$.

Remember:
A **positive** answer means that the area is **above** the x-axis and a **negative** answer means that the area is **below** the x-axis.

If there is a mixture (above and below) you would need to find each area separately and then add the areas (ignoring the negative sign).

e.g. to find the area enclosed by the curve $y = x^3 - 7x^2 + 10x$ and the x-axis:



You would integrate with the limits 0 and 2 then **separately** integrate with the limits 2 and 5 (expect a negative answer as this area is below the line).

Total area: ignore the negative sign and add the 2 amounts together.

Try it – you should get an area of $\frac{253}{12}$ (approx. 21.1)