Summary sheet: Graphs and transformations

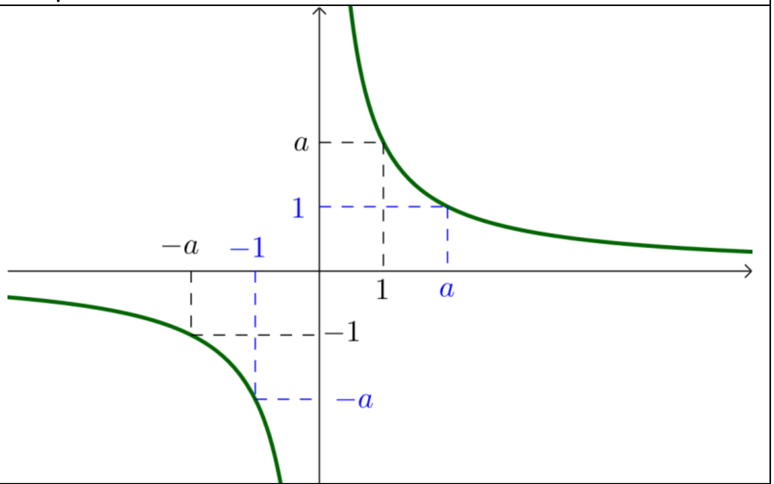
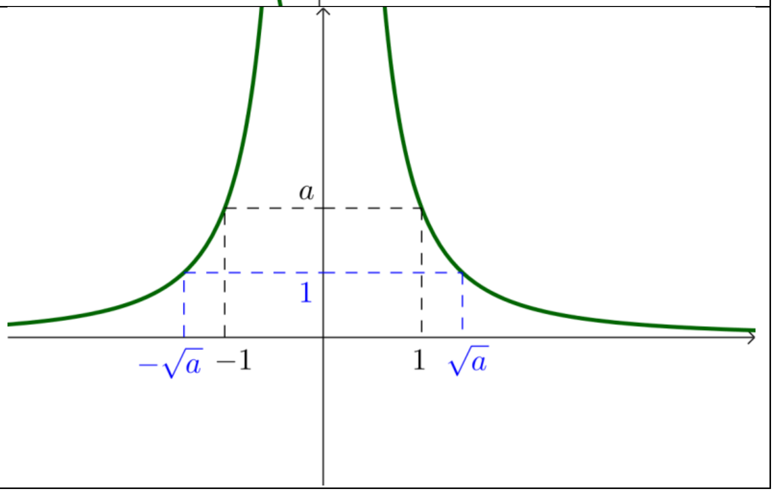
B7 Understand and use graphs of functions; sketch curves defined by simple equations including polynomials, y = a/x and y = a/x^2 (including their vertical and horizontal asymptotes); interpret algebraic solution of equations graphically; use intersection points of graphs to solve equations
Understand and use proportional relationships and their graphs
B9 Understand the effect of simple transformations on the graph of y = f (x) including sketching associated graphs: y = af(x), y = f(x + a), y = f(x) + a, y = f(ax)

Graphs of functions

Functions or equations show the relationship between x and y and allow you to plot (or sketch) a graph. You might recognise some graphs straight away, and easily sketch them, but if not just find a few points (coordinates) and plot them to get an idea of what the graph looks like.

Think about the shape of the graph, where it crosses the axes and whether there are any asymptotes.

e.g. Think about the points shown on these 2 graphs and also where the graphs would be above and below each other if you plotted them on the same axes.

Equation	Think about	Graphs
$y = \frac{a}{x}$	**Asymptote** at $\boldsymbol{x = 0}$ (because you can't divide by 0) and at $\boldsymbol{y = 0}$ (because $a \div x \neq 0$). When x is positive, y is positive, and when x is negative, y is negative. You could think what this looks like by letting $a = 3$ (or any number) & plotting a few points.	
$y = \frac{a}{x^2}$	**Asymptote** at $\boldsymbol{x = 0}$ (because you can't divide by 0) and at $\boldsymbol{y = 0}$ (because $a \div x \neq 0$). When x is positive, y is positive, and when x is negative, y is positive. You could think what this looks like by letting $a = 3$ (or any number) & plotting a few points.	

For help with sketching polynomials see 'Polynomials summary sheet'.

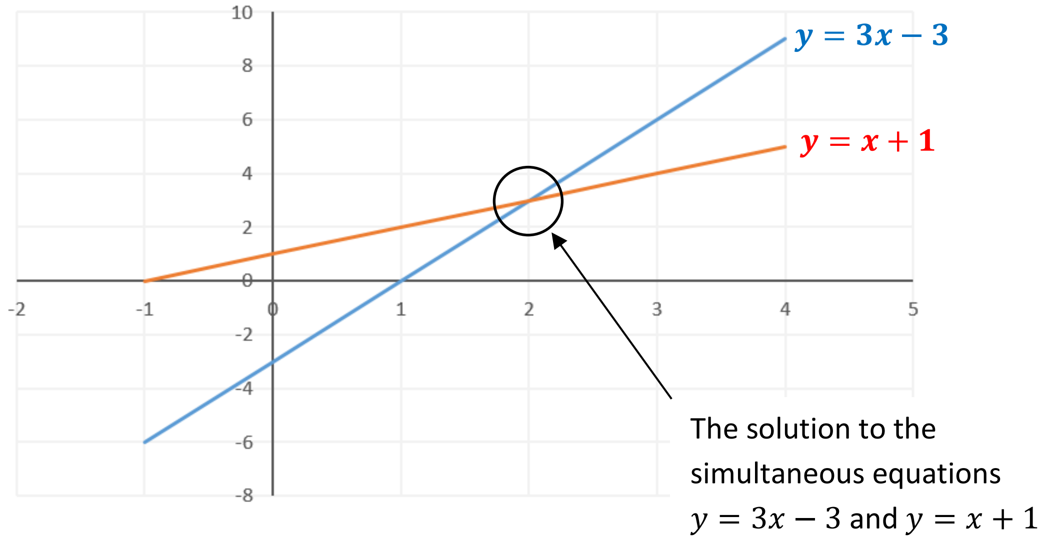
Summary sheet: Graphs and transformations

Interpret algebraic solution of equations graphically

When you have solved 2 (or more) equations simultaneously you can then plot the graphs, on the same axes, and show the intersection point (i.e. your solution) on the graphs.

Use intersection points of graphs to solve equations

You can plot 2 graphs on the same axes, and the intersection of the graphs will be the solution to the simultaneous equations.



Understand and use proportional relationships and their graphs

Two quantities are proportional if they vary in the same way (e.g. if one doubles, the other doubles). You can draw a graph to show the relationship. The easy way to decide on the graph is to put a k in front of the x part of the equation.

e.g. $\boldsymbol{y \propto x}$ will become $\boldsymbol{y = kx}$ (a straight line graph with a gradient of k)
(y is proportional to x)

e.g. $\boldsymbol{y \propto x^2}$ will become $\boldsymbol{y = kx^2}$ (a curve (a larger value of k makes the curve steeper))
(y is proportional to x^2)

e.g. $\boldsymbol{y \propto \frac{1}{x}}$ will become $\boldsymbol{y = \frac{k}{x}}$ (a curve – shown on the 1st page)

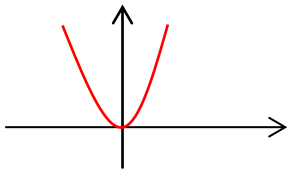
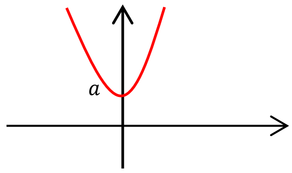
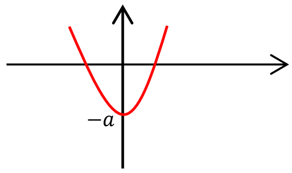
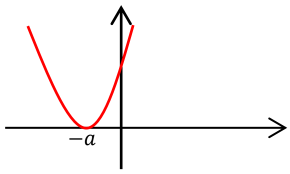
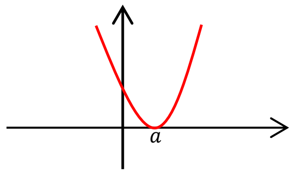
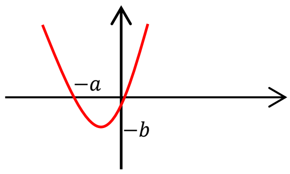
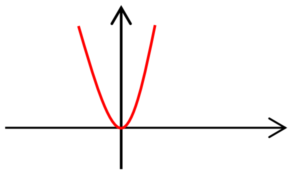
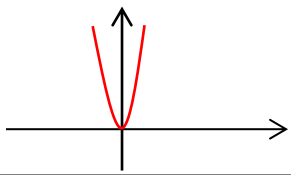
This is called inverse proportion because as one value gets bigger the other gets smaller

Remember that for any of these graphs you can find k by substituting a point (x, y) and rearranging.

Summary sheet: Graphs and transformations

Transformations

You need to understand what happens to graphs when they are transformed. The following gives examples of some graph transformations that you need to learn:

e.g. graph: **$y = x^2$**	Original	
$y = x^2 + a$	Translation a units in *y* direction	
$y = x^2 - a$	Translation $-a$ units (i.e. down) in *y* direction	
$y = (x + a)^2$	Translation $-a$ (i.e. to the left) in *x* direction	
$y = (x - a)^2$	Translation a (i.e. to the right) in *x* direction	
$y = (x + a)^2 - b$	Translation $-a$ units in the *x*-direction and $-b$ units in the *y*-direction	
$y = ax^2$	Stretch scale factor a parallel to the *y*-axis	
$y = (ax)^2$	Stretch scale factor $\frac{1}{a}$ parallel to the *x*-axis	

Remember that these transformations work for all graphs, $y = x^2$ is just an example.
Try some yourself to see what happens. Replace a with a number and see how it affects your graph.