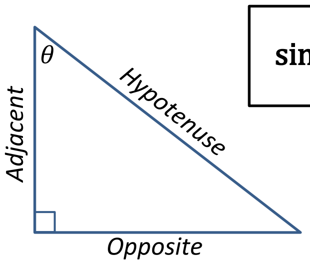
Summary sheet: Trigonometry

E1 Understand and use the definitions of sine, cosine and tangent for all arguments; the sine and cosine rules; the area of a triangle in the form ½bc sin A
E3 Understand and use the sine, cosine and tangent functions; their graphs, symmetries and periodicity
E5 Understand and use tan θ = sin θ / cos θ
Understand and use $\sin^2\theta + \cos^2\theta = 1$
E7 Solve simple trigonometric equations in a given interval, including quadratic equations in sin, cos and tan and equations involving multiples of the unknown angle

Sine, cosine and tangent

Sine (sin), cosine (cos) and tangent (tan) can be used to find missing sides or angles of right angled triangles. You will need to remember the definitions; some people like to remember **SOHCAHTOA**.



$$\sin\theta = \frac{opposite}{hypotenuse} \qquad \cos\theta = \frac{adjacent}{hypotenuse} \qquad \tan\theta = \frac{opposite}{adjacent}$$

Remember: The hypotenuse is the longest side, the adjacent is next to (or touching) the angle and the opposite is at the other side of the triangle, not touching the angle.

Finding an angle:

e.g. find θ when $\sin\theta = 0.75$

$$\theta = \sin^{-1}(0.75)$$
$$\theta = 48.6°$$

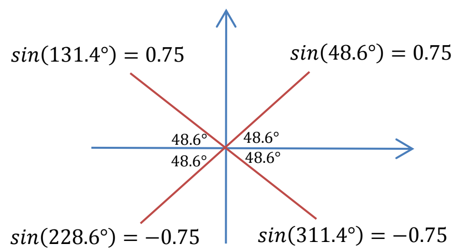
Remember that if you are looking for all the angle between 0 and 360^0 you will need to think about the CAST diagram to work out the other angle where $\sin\theta = 0.75$. Remember that, using symmetry, there is a matching angle in all 4 quadrants but it will only be positive in 2 of the quadrants.

The CAST diagram tells you where the trig ratios are positive:

S	**A**
T	**C**

Sin positive	**A**ll positive
Tan positive	**C**os positive

e.g.



Two trig identities to learn and remember:

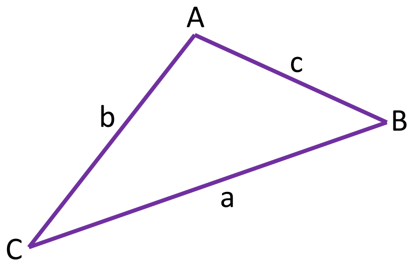
$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$ Useful when solving equations containing both $\sin\theta$ and $\cos\theta$.

$$\sin^2\theta + \cos^2\theta = 1$$ Useful when solving quadratic equations containing both $\sin\theta$ and $\cos\theta$

Summary sheet: Trigonometry

The sine and cosine rule

The sine and cosine rule can be used to find missing sides or angles for non-right angled triangles. For both rules you need to label your angles A, B, C then label the side opposite each angle with the same letter (in lower case).



The sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

This is the easiest when finding a side.

can also be written as

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

This is the easiest when finding an angle.

Use the sine rule when you: **know** 2 sides and 1 angle (not between the 2 sides)
want an angle or a side

The cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

This is the easiest when finding a side.

can be rearranged to

$$\text{cosc}A = \frac{b^2+c^2-a^2}{2bc}$$

This is the easiest when finding an angle.

Use the cosine rule when you: **know** 2 sides and the angle between them
want the 3rd side

OR **know** 3 sides
want an angle

Tip: if you notice the pattern of a's, b's and c's then it's easy to write the cosine rule any way round.

Same letter at beginning and end

e.g. $b^2 = \underbrace{a^2 + c^2}_{} - 2ac \cos B$

These 2 letters repeated here

The area of a triangle

To find the area of a triangle you need to know 2 sides and the angle between them.

Area of a triangle = $\frac{1}{2}bc \sin A$

Summary sheet: Trigonometry

The graphs of $\sin x$, $\cos x$ and $\tan x$

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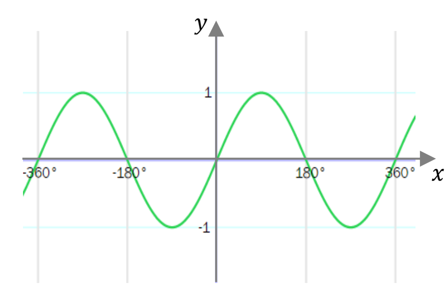
<td>
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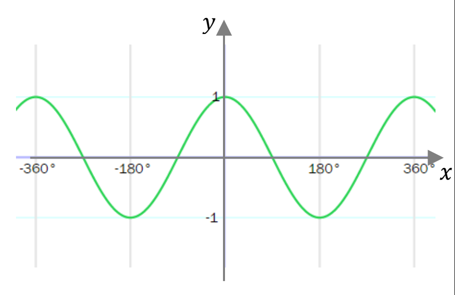
<td>
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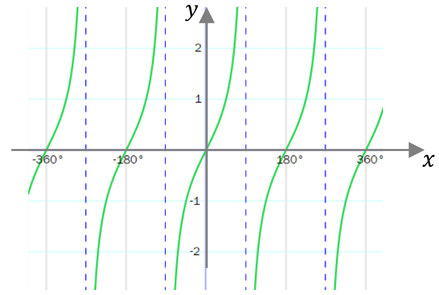
<td>
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Period = 360°
Rotational symmetry about the origin
Lies between -1 and 1
Line of symmetry at $x = 90°$ and $x = -90°$

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Period = 360°
Line of symmetry at y-axis
Lies between -1 and 1
$y = \cos x$ is the same as $y = \sin x$ shifted left by 90°

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Period = 180°
Rotational symmetry about the origin
Lies between $-\infty$ and ∞
Asymptotes at $x = \pm 90^{\circ}, \pm 270°, \pm 450°$ etc

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Solving trig equations

Trig equations are all different and the best way to learn them is to practice as many as you can.

Some techniques to help you:

Technique	**Example:**
Rearrange to make $\sin\theta$, $\cos\theta$ or $\tan\theta$ the subject	$\sin\theta - 0.3 = 0$ $\sin\theta = 0.3$ $\theta = 17.5°$
Factorise if possible	$3\cos\theta\sin\theta + \sin\theta = 0$ $\sin\theta(3\cos\theta + 1) = 0$ $\sin\theta = 0 \;\; or \;\; 3\cos\theta + 1 = 0$ $\theta = 0° \;\text{ or }\; \theta = 109.5°$
If it's a mixture of $sin\theta$ and $cos\theta$ check if the identity $tan\theta = \frac{sin\theta}{cos\theta}$ will help.	$\sin\theta = 4\cos\theta$ $\dfrac{\sin\theta}{\cos\theta} = 4$ $\tan\theta = 4$ $\theta = 76°$
If you have a mixture of $sin\theta$ and $cos\theta$ in a quadratic, check if the identity $sin^2\theta + cos^2\theta = 1$ helps, then factorise or use the quadratic formula.	$\cos^2\theta = \sin\theta + 1$ $(\cos^2\theta = 1 - \sin^2\theta) \rightarrow$ (substitute in) $1 - \sin^2\theta = \sin\theta + 1$ $\sin^2\theta + \sin\theta = 0$ Factorise and solve
Solve multiples of the unknown angle:	$\tan(2\theta) = 5$ $2\theta = \tan^{-1}(5)$ $2\theta = 78.7°$ $\theta = 39.4°$

Remember for all of the above to think about the CAST diagram and make sure that you have found all the angles within the range asked for. Also remember that with multiples of the unknown angle you can go around the circle more than once, e.g. if $0° \le \theta \le 360°$ then $0° \le 2\theta \le 720°$.

