Summary sheet: Coordinate geometry

C1 Understand and use the equation of a straight line, including the forms $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$; gradient conditions for two straight lines to be parallel or perpendicular. Be able to use straight line models in a variety of contexts

C2 Understand and use the coordinate geometry of the circle including using the equation of a circle in the form $(x - a)^2 + (y - b)^2 = r^2$; completing the square to find the centre and radius of a circle; use of the following properties:

- the angle in a semicircle is a right angle
- the perpendicular from the centre to a chord bisects the chord
- the radius of a circle at a given point on its circumference is perpendicular to the tangent to the circle at that point

The equation of a straight line

There are various ways of writing the equation of a straight line. Remember that however you write it, it will contain an x and a y (or 2 different variables), because the equation is telling you the link between the variables.

$y = mx + c$	The most commonly used equation of a straight line, where m is the gradient and c is the y-intercept.
$y - y_1 = m(x - x_1)$	Useful if you are asked to find the equation of a straight line given the gradient and a point on the line. Substitute the gradient and the point and rearrange. N.B. if you find this equation difficult to remember you can always substitute the gradient and the point into the first equation and rearrange to find c.
$ax + by + c = 0$	Another way of writing a straight line. Useful when the gradient is a fraction. E.g. $y = \frac{2}{3}x + 4$ can be rearranged to $3y - 2x - 4 = 0$

Parallel or perpendicular

To decide whether 2 lines are parallel (like train tracks) or perpendicular (a right angle) you would look at the gradients.

Parallel	$m_1 = m_2$	The gradients are the same
Perpendicular (right angle)	$m_1 m_2 = -1$	Substitute the known gradient in and rearrange to find the perpendicular OR an easy way to find a perpendicular gradient is to think: *Turn it upside down and use the opposite sign*. e.g. gradient = 3, perpendicular gradient = $-\frac{1}{3}$

Finding the distance between 2 points

Use Pythagoras: Distance (length) = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Find the coordinates of the midpoint of a line

Midpoint = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

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Form the equation of a straight line

Find the gradient (using $m = \frac{y_2 - y_1}{x_2 - x_1}$) then substitute the gradient and a given point into one of the forms shown above to find c.

Sketching a straight line

Sometimes you might be asked to sketch a straight line. If you are given 2 points on the line just plot them and join them together with a straight line.

$y = mx + c$	Plot the y-intercept then sketch the graph (remember to consider whether the gradient is positive or negative and how steep it should look. A gradient of 1 is at a $45°$ angle).
$ax + by + c = 0$	If you are given the equation in this form, it's probably easiest just to find the x and y intercepts. Remember the **x-intercept is when $y = 0$** and the **y-intercept is when $x = 0$**

Find the point of intersection of 2 lines

At a point of interception, the x and y coordinates are the same for both lines so you would solve them simultaneously (see 'Equations and Inequalities Summary Sheet' for how to do this).

The equation of a circle

The general equation of a circle is: $\boldsymbol{(x - a)^2 + (y - b)^2 = r^2}$

Once you know this it is easy to find the centre and the radius.

The centre is at (a, b) and the radius = r

e.g. Consider the circle $(x - 3)^2 + (y + 2)^2 = 9$

The centre is at $(3, -2)$ and the radius = 3

e.g. Consider the circle $x^2 + y^2 = 16$

The centre is at $(0, 0)$ and the radius = 4

This is a circle with the centre at the origin because $a = 0$ and b$= 0$

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Complete the square to find the centre and the radius

The equation of a circle could be given in a different form. To find the centre and the radius you would need to complete the square.

e.g. Find the centre and the radius of the following circle: $\boldsymbol{x^2 + y^2 + 10x - 4y = -4}$

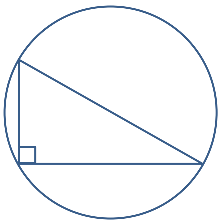
	$x^2 + y^2 + 10x - 4y = -4$
Collect x's and y's together:	$x^2 + 10x + y^2 - 4y = -4$
	Half this (10) … Half this (4)
Complete the square:	$(x + 5)^2 - 5^2 + (y - 2)^2 - 2^2 = -4$
	Subtract back off … Subtract back off
Collect the numbers at the right hand side:	$(x + 5)^2 + (y - 2)^2 = -4 + 5^2 + 2^2$
Tidy up:	$(x + 5)^2 + (y - 2)^2 = 25$

In this form it is easy to see the centre and the radius of the circle. Centre = $(-5, 2)$ and radius = 5.

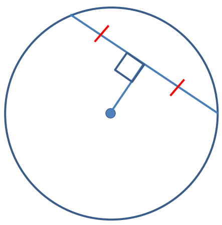
Circle properties

There are 3 properties of circles that you need to remember.

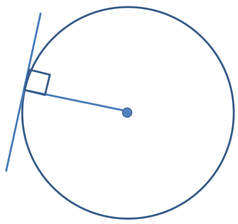
- The angle in a semi-circle is a right angle.



- If you draw a line from the centre of the circle, perpendicular to a chord, then the line will bisect the chord.



- Any tangent to a circle is perpendicular to the radius at the point where it touches the circle.



Find the point of intersection of a line and a circle

Solve simultaneously – remember that when a curve is involved (i.e. there are powers) it is easiest to use the method of substitution (see 'Equations and Inequalities Summary Sheet' for help).