Summary sheet: Equations and inequalities

B4 Solve simultaneous equations in two variables by elimination and by substitution, including one linear and one quadratic equation

B5 Solve linear and quadratic inequalities in a single variable and interpret such inequalities graphically, including inequalities with brackets and fractions
Express solutions through correct use of 'and' and 'or', or through set notation
Represent linear and quadratic inequalities such as y > x + 1 and y > ax^2 + bx + c graphically

Simultaneous equations

Simultaneous equations are equations where the solutions are pairs of values of x and y that satisfy both equations. You can solve them by elimination or by substitution.

Solving using elimination

Elimination means adding or subtracting the equations to eliminate one of the unknowns. These are the steps to follow:

- Make the coefficients of one of the unknowns the same. *(whichever seems easier)*
- Add or subtract the equations to eliminate one unknown (remember: **same** signs **subtract**, **different** signs **add**).
- Solve the new equation to find the first unknown.
- Substitute back into one of the original equations to find the other unknown.
- Check your values of x and y in both equations.

Example:

- **Solve simultaneously:** $\boldsymbol{3x - 2y = 4}$ (equation 1)
  $\boldsymbol{2x + 5y = 9}$ (equation 2)

Make the coefficient of x the same:	$\boldsymbol{6x - 4y}$	$=$	$\boldsymbol{8}$	(3)	*(equation 1)x2*
	$\boldsymbol{6x + 15y}$	$=$	$\boldsymbol{27}$	(4)	*(equation 2)x3*
Subtract ($x's$ are the same sign):	$\boldsymbol{-19y}$	$=$	$\boldsymbol{-19}$		*(3) – (4)*
Solve to find y:	$\boldsymbol{y}$	$=$	$\boldsymbol{1}$		
Substitute into (2) to find x:	$\boldsymbol{2x + 5(1)}$	$=$	$\boldsymbol{9}$		*Replace y with 1*
	$\boldsymbol{x}$	$=$	$\boldsymbol{2}$		*Solve to find x*

Checks:

Equation (1):	$3x - 2y = 4$	$3(2) - 2(1) = 4$	$6 - 2 = 4$	✓
Equation (2):	$2x + 5y = 9$	$2(2) + 5(1) = 9$	$4 + 5 = 9$	✓

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Solving using substitution

Substitution means substituting one of the equations into the other to get rid of one of the unknowns.
These are the steps to follow (x and y have been used but it could be any letters):

- Rearrange one of the equations (if necessary) to make either **x or y the subject**.
- **Substitute** into the other equation.
- Solve the new equation to find x or y.
- Substitute back into your rearranged equation to find the value of the other letter.
- Check your values of x and y in both equations

Example:

- **Solve simultaneously:** $\mathbf{3y - 2x = 12}$ (equation 1)
  $\mathbf{y + 5x = -13}$ (equation 2)

Rearrange equation 2: $\mathbf{y = -5x - 13}$ *(to make y the subject)*

Substitute y into equation 1: $\mathbf{3(-5x - 13) - 2x = 12}$ *(Replace y with $-5x - 13$)*

Tidy up and solve for x:

$$\begin{aligned} -15x - 39 - 2x &= 12 \\ -17x &= 51 \\ \underline{\underline{x}} &\underline{\underline{= -3}} \end{aligned}$$

Substitute x to find y: *(Use rearranged version)*

$$\begin{aligned} y &= -5x - 13 \\ y &= -5(-3) - 13 \\ \underline{\underline{y}} &\underline{\underline{= 2}} \end{aligned}$$

Checks:

Equation (1):	$3y - 2x = 12$	$3(2) - 2(-3) = 12$	$6 + 6 = 12$	✓
Equation (2):	$y + 5x = -13$	$2 + 5(-3) = -13$	$2 - 15 = -13$	✓

One linear and one quadratic equation

When you have one linear and one quadratic equation the best way to solve them is by substitution.

e.g. Solve the simultaneous equations:

$$\mathbf{x + y = 8}$$
$$\mathbf{x^2 + y^2 = 34}$$

Rearrange the linear equation to make x or y the subject then substitute it into the quadratic equation.
Try it! You should get the answers $x = 3$ and $y = 5$ OR $x = 5$ and $y = 3$

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Linear inequalities

An inequality can use any of these signs: $<$ $>$ $\leq$ $\geq$ instead of the = sign. This means that your answer will be a range of values instead of just one value. You can solve them in the same way as you would if you had an = sign but there is one thing you need to remember:

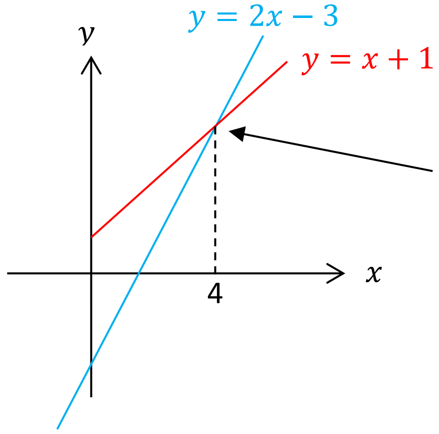
if you multiply or divide by a negative number you need to reverse the inequality sign.

e.g. Solve the inequality $2x-3>x+1$ and sketch the outcome on a graph.

Calculation:

$$2x-3>x+1$$
$$2x-x>1+3$$
$$x>4$$

Graph:



This is the point where $2x-3$ becomes greater than (above) $x+1$

e.g. Solve the inequality: $\mathbf{5-x\geq 2x-1}$

$$-3x\geq -6$$
$$x\leq 2$$

Notice that the inequality is reversed as you have divided by -3

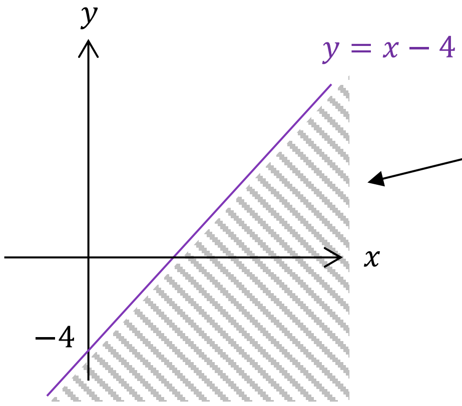
e.g. Solve the inequality: $\dfrac{x+3}{2}<\dfrac{x-2}{5}$

$$5(x+3)<2(x-2)$$
$$5x+15<2x-4$$
$$3x<-19$$
$$x<-\frac{19}{4}$$

With all inequalities don't forget to check that your answer works! Try a value above and below your answer and check that the inequality is correct.

e.g. Sketch the inequality: $\mathbf{y<x-4}$

For this type of question sketch the graph of $y=x-4$ then show whether you want the area above or below the line.



In this case you want the area below the line because y is less than (<) the line.

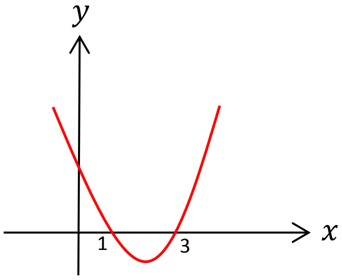
Quadratic inequalities

To solve a quadratic inequality: do a quick sketch (you will need to know the shape and the roots) then look for the appropriate part of the graph (i.e. <0 (below the x-axis) or >0 (above the x-axis) depending on what you are looking for).

e.g. Solve the inequality: $\boldsymbol{x^2 - 4x + 3 > 0}$

Find the roots by setting the quadratic equal to 0, then do a quick sketch:

$$x^2 - 4x + 3 = 0$$
$$(x-1)(x-3) = 0$$
$$x = 1 \text{ or } x = 3$$



You want to know where the quadratic is > 0 so, looking at the graph you can see that the curve is above the x-axis **when $\boldsymbol{x < 1}$ or when $\boldsymbol{x > 3}$**.
(Notice that there are 2 separate parts to this answer because there are 2 separate parts of the line that are above the x-axis.)

If the inequality sign had been the other way round:
e.g. Solve the inequality: $\boldsymbol{x^2 - 4x + 3 \leq 0}$

Follow the same process but now you want to know when the quadratic is ≤ 0 (on or below the x-axis) so, looking at the graph, you can see that this is when x is between 1 and 3, so $\boldsymbol{1 \leq x \leq 3}$.

e.g. Sketch the inequality: $\boldsymbol{y > x^2 - 5x + 6}$
Sketch the graph of $y = x^2 - 5x + 6$ then show whether you want the area above or below the curve.