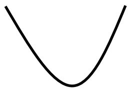
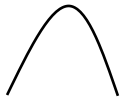
Summary sheet: Quadratic functions

B3 Work with quadratic functions and their graphs; the discriminant of a quadratic function, including the conditions for real and repeated roots; completing the square; solution of quadratic equations including solving quadratic equations in a function of the unknown

The general shape of quadratic graphs

$y = x^2$  $y = -x^2$ 

Solving quadratic equations

The two most common ways to solve a quadratic equation are **factorising** and using the **formula**. They will both give you the same answer so just use the most appropriate method.

Remember that to solve a quadratic equation you should collect all the terms on one side so that the other side of the equation is 0. When you solve the equation, it you have found the **roots** (i.e. where the graph of the quadratic function **crosses the x-axis**).

Solving by factorising

This means factorising the quadratic into 2 brackets.

e.g. Solve $x^2 + 6x + 8 = 0$ using factorisation.

$$x^2 + 6x + 8 = 0$$
$$(x + 4)(x + 2) = 0$$

Remember you need 2 numbers that multiply to give 8 and add to give 6.

Therefore: $\boldsymbol{x = -4}$ or $\boldsymbol{x = -2}$

Solving by using the formula

The quadratic formula:

When $y = ax^2 + bx + c$ $\qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

e.g. Solve $x^2 + 6x + 7 = 0$ using the formula.

First of all, make a note that: $a = 1$
$b = 6$
$c = 7$

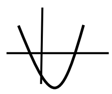
then substitute into the formula (put each number in a bracket so that you are careful to get the correct sign):

$$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(1)(7)}}{2(1)}$$

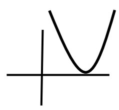
$$x = \frac{-6 \pm \sqrt{8}}{2} = -3 \pm \sqrt{2}$$

The Discriminant:
The expression inside the square root sign is called the discriminant and tells you what type of roots to expect.

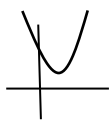
If $\boldsymbol{b^2 - 4ac > 0}$ there are **2 real roots**
(i.e. the curve crosses the x-axis in 2 places)



If $\boldsymbol{b^2 - 4ac = 0}$ there is **1 real root**
(i.e. the curve touches the x-axis in 1 place)



If $\boldsymbol{b^2 - 4ac < 0}$ there are **no real roots**
(i.e. the curve does not cross the x-axis)



Innovators in Mathematics Education

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Completing the square to find the turning point

Remember that the completed square version of a quadratic equation looks like this:

$$\boldsymbol{a(x+b)^2+c=0}$$

The turning point (maximum or minimum) will be at $(-\boldsymbol{b}, \boldsymbol{c})$

e.g. Find the turning point of the equation $\boldsymbol{(x+5)^2-7=0}$

The turning point (minimum) will be at $(-5,-7)$

e.g. Find the turning point of the equation $\boldsymbol{5-3(x-4)^2=0}$

The turning point (maximum) will be at $(4,5)$
Notice that the turning point is a maximum because the x^2 term is negative.

Don't forget:
If you need to solve the quadratic to find the roots and it is already in the completed square form, you don't need to factorise or use the formula you can just rearrange to find x.

Solving equations in a function of the unknown

Sometimes you may need to solve an equation which is a 'disguised quadratic'. This is an equation whch involves one term in which the power of x is twice the power of x in another term.

e.g. Solve the equation $\boldsymbol{x^4+3x^2+2=0}$

The equation could be rewritten as $\boldsymbol{(x^2)^2+3x^2+2=0}$
So you can let $y=x^2$ and you have: $y^2+3y+2=0$
Now you can solve by factorising.

e.g. Solve the equation $\boldsymbol{x^6+5x^3+6=0}$

The equation could be rewritten as $\boldsymbol{(x^3)^2+5x^3+2=0}$
So you can let $y=x^3$ and you have: $y^2+5y+6=0$
Now you can solve by factorising.