Summary sheet: Indices and surds

B1 Understand and use the laws of indices for all rational exponents (fraction powers)
B2 Use and manipulate surds, including rationalising the denominator

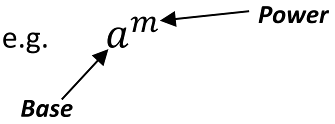
Indices (powers)

An index (power) tells you how many times to multiply something by itself:

e.g. x^5 means $x \times x \times x \times x \times x$

There is a base and a power:

e.g. a^m



There are a few rules of indices that you need to learn and remember how to use:

Rule	Meaning	Example
$\boldsymbol{a^m \times a^n = a^{m+n}}$	To **multiply** 2 numbers with the same base you **add** the powers.	$5^3 \times 5^4 = 5^7$
$\boldsymbol{\frac{a^m}{a^n} = a^{m-n}}$	To **divide** 2 numbers with the same base you **subtract** the powers.	$\frac{3^7}{3^2} = 3^5$
$\boldsymbol{(a^m)^n = a^{m\times n}}$	To simplify a power inside and outside of a **bracket** you **multiply** the powers.	$(6^4)^3 = 6^{12}$
$\boldsymbol{a^{-m} = \frac{1}{a^m}}$	A negative power means **"one over"** so send everything to the bottom of a fraction.	$8^{-5} = \frac{1}{8^5}$
$\boldsymbol{a^{\frac{m}{n}} = (\sqrt[n]{a})^m}$	A fractional power means a **root**. The bottom of the fraction tells you the root and the top tells you the power.	$8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = 2^2 = 4$
$\boldsymbol{a^0 = 1}$	**Anything** to the power of zero = 1	$57^0 = 1$

Finally
Remember that any number to the power of one stays the same: e.g. $72^1 = 72$

And 1 to the power of anything is 1: e.g. $1^{15} = 1$
$1^{-356} = 1$

Summary sheet: Indices and surds

Surds (roots)

A surd (root) is the inverse of a power:
e.g. $\sqrt{25}$ means "which number multiplied by itself would give 25?" the answer is 5 because 5x5=25.
Remember that a surd can be part rational, e.g. $(3+\sqrt{7})$ has a rational part (3) and the root part $(\sqrt{7})$.

It is a good idea to remember the first few perfect square numbers so that you can spot them when you are working with surds, (**1** (1x1), **4** (2x2), **9** (3x3), **16** (4x4), **25** (5x5), **36** (6x6) etc).

Writing surds in their simplest form

If a square root has a perfect square number as a factor, then it can be simplified.

e.g. $\sqrt{20}$ can be re-written as $\sqrt{4} \times \sqrt{5}$ which simplifies to $2\sqrt{5}$

Perfect square

Adding and subtracting surds

Remember to add or subtract **like terms** (i.e. the rational numbers and the roots (of the same number))

e.g. $(\mathbf{7}+\mathbf{3\sqrt{2}})+(\mathbf{8}-\mathbf{\sqrt{2}}) = \mathbf{15}+\mathbf{2\sqrt{2}}$

Add rational parts: $(7+8=\mathbf{15})$
Add roots: $(3\sqrt{2}-1\sqrt{2}=\mathbf{2\sqrt{2}})$

Multiplying surds

If there is no rational part then multiplying is easy: e.g. $\sqrt{3}\times\sqrt{5} = \sqrt{15}$

If there is a rational part then multiply out the brackets (either using FOIL (first, outside, inside, last) or the smile – whichever you prefer):

e.g. $(\mathbf{5}+\mathbf{\sqrt{3}})\times(\mathbf{2}-\mathbf{\sqrt{3}}) = 10-5\sqrt{3}+2\sqrt{3}-\sqrt{3}\sqrt{3}$ tidies up to give $\mathbf{7-3\sqrt{3}}$

Remember that $\sqrt{3}\times\sqrt{3}=3$

Rationalising the denominator

You rationalise the denominator to get rid of the surd on the bottom of a fraction. To rationalise the denominator just multiply the top and bottom of the fraction by the **bottom of the fraction with the opposite sign in front of the root**.

e.g. Simplify $\frac{\mathbf{3+\sqrt{5}}}{\mathbf{2-\sqrt{5}}}$ **by rationalising the denominator.**

Remember the rule: multiply the top and bottom of the fraction by the **bottom of the fraction with the opposite sign in front of the root.**

$$\frac{\mathbf{3+\sqrt{5}}}{\mathbf{2-\sqrt{5}}}\times\frac{\mathbf{2+\sqrt{5}}}{\mathbf{2+\sqrt{5}}} = \frac{\mathbf{6+3\sqrt{5}+2\sqrt{5}+\sqrt{5}\sqrt{5}}}{\mathbf{4+2\sqrt{5}-2\sqrt{5}-\sqrt{5}\sqrt{5}}} = \frac{\mathbf{11+5\sqrt{5}}}{\mathbf{-1}} = \mathbf{-11-5\sqrt{5}}$$

Notice these are the same – but the sign in front of the root has changed.

Changing the sign in front of the root makes the middle parts cancel each other out & disappear.