Section 1: Solving problems

Notes and Examples

These notes contain subsections on

- Solving mathematical problems
- Mathematical modelling

Solving mathematical problems

To develop good problem-solving skills it's useful to practise the following:

- problems given to you 'in words' so that you need to formulate the problem in a way which allows mathematical methods to be used
- problems that use the methods you know but in ways you may not spot straight away
- problems where the solution is in several stages but you are not given any guidance about what those stages might be

Some examples are given below. Here is a problem given 'in words'.

Example 1

Freddie and Louis each start with the same number of sweets. After Freddie bought 15 more and Louis ate 5 they had 40 sweets between them. How many sweets did they each have initially?

Solution

Let x be the number of sweets that Freddie starts with.

Let y be the number of sweets that Louis starts with.

In 'word' problems you often need to introduce some variables to represent the quantities involved.

Here the variable names x and y have been chosen.

The information "*Freddie and Louis each start with the same number of sweets*" means that

$$x = y \quad (*)$$

The information given in the question is used to form equations involving the variables.

The information "*After Freddie bought 15 more and Louis ate 5 they had 40 sweets between them*" means that

$$(x + 15) + (y - 5) = 40$$

This simplifies to

$$x + y = 30 \quad (**)$$

Using equation (*) in (**) gives

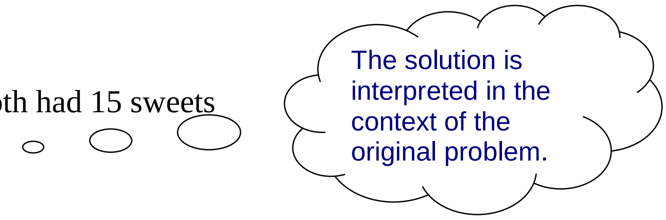
The equations are solved.

$$2x = 30$$
$$\Rightarrow x = 15$$

Then equation (*) tells you that

$$y = 15.$$

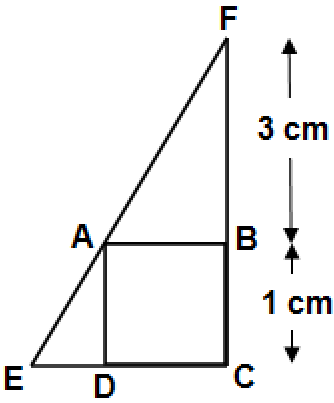
This means that Louis and Freddie both had 15 sweets to begin with.



Here is a problem that uses a technique you will be familiar with (similar triangles) but in a way that you may not spot straight away. The solution is also in several stages.

Example 2

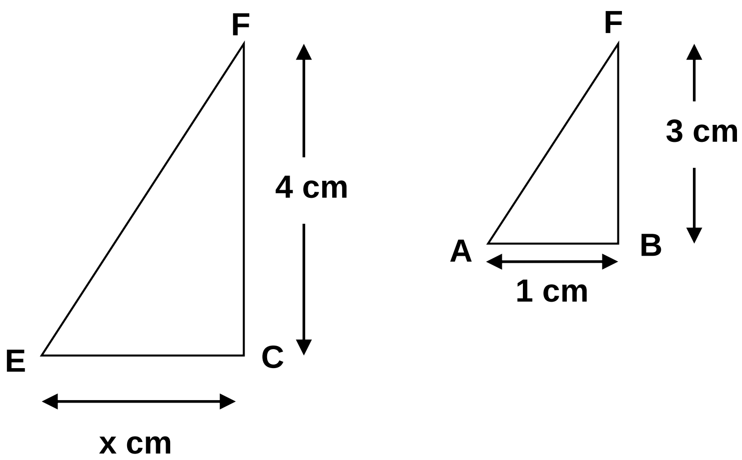
In the diagram ABCD is a square. What is the area of the triangle EFC?



Solution

Stage 1 – Pick out two similar triangles and use this to calculate a useful side length.

Below are two triangles from the diagram, drawn separately. AB has length 1cm because it is another side of the square ABCD.



Since the two triangles share the angle at F and are right-angled, they must be similar triangles (all three angles are the same).

Therefore $\frac{x}{1}=\frac{4}{3}\Rightarrow x=\frac{4}{3}$.

Stage 2 – use the information you now have to calculate the area required.

Now you have the length of the base of triangle EFC and its height. These are $\frac{4}{3}$ cm and 4 cm respectively.

Therefore the area of EFC is $\frac{1}{2}\times\frac{4}{3}\times 4=\frac{16}{6}=\frac{8}{3}\text{cm}^2$

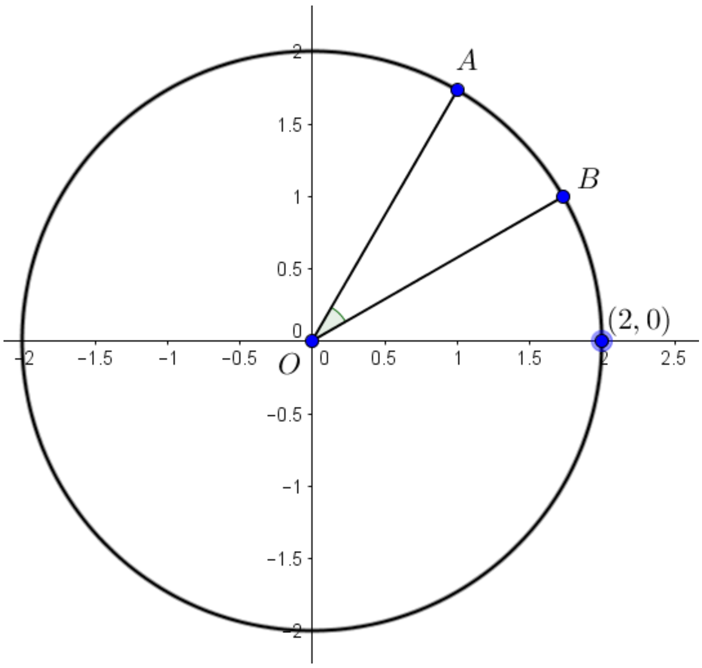
Here is a problem where the solution is in several stages and which uses a familiar technique (trigonometry) but in a way you may not spot straight away.

Example 3
The centre of the circle shown is the origin.

The x-coordinate of A is 1.

The y-coordinate of B is 1.

Calculate the marked angle.



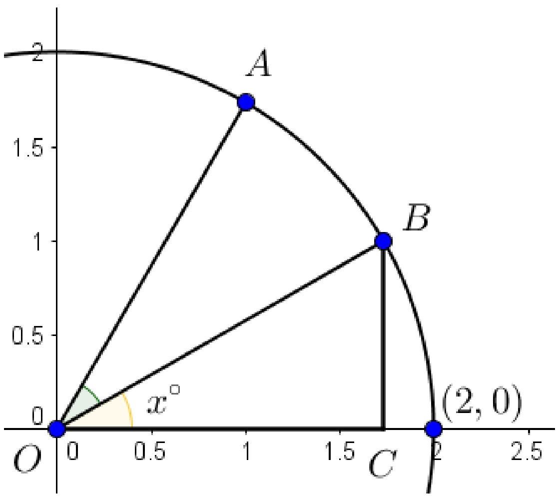
Solution

Stage 1 – Form a right angled triangle to work out other useful angles.

It would be useful to know the value of x in the diagram on the right. You can form a right angled triangle including this angle to do this.

Since the circle goes through (2,0) its radius is 2.

Draw a vertical line from B down to the y-axis as in the diagram on the

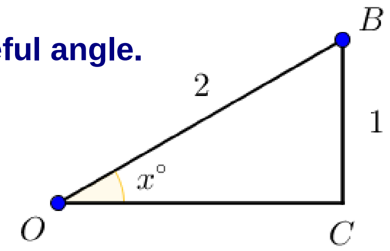


right. The point where it meets the y-axis has been labelled as C.

The angle OCB is a right angle, since BC is vertical and OC is horizontal. Since the y-coordinate of B is 1, its length is 1.

The length of OB is 2 since this is a radius of the circle.

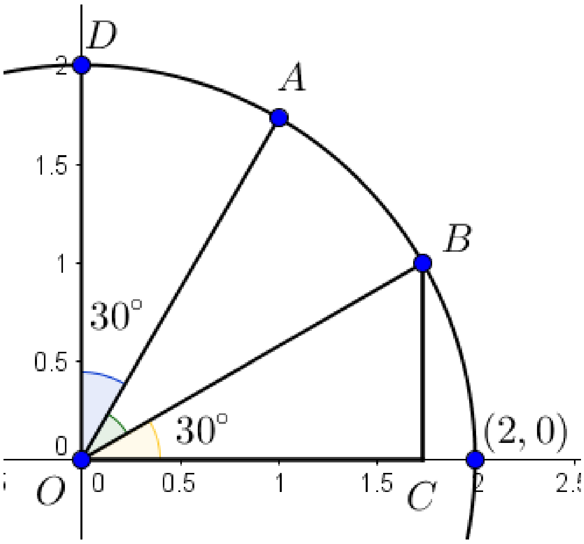
Stage 2 – use trigonometry to work out a useful angle.



The value of x can now be calculated as follows.

$$\sin x = \frac{1}{2} \Rightarrow x = 30°$$

Stage 3 – use symmetry to get another useful angle and then complete the problem.



By symmetry (because the x-coordinate of A is 1) the angle AOD in the diagram on the right is also 30°.

Angles AOD, AOB and BOC sum to 90°.

Therefore angle AOB is also 30°.

Mathematical modelling

Solving problems using mathematics is an important skill in almost all areas of life! When you need to solve a real-life problem involving mathematics, the first step is to express the problem in mathematical terms. This usually involves making simplifying assumptions.

The difficulty with applying a mathematical model to a real-life situation is that it may not be immediately obvious whether the assumptions are realistic or not. It is usually helpful to have some kind of experimental result to refer to.

As part of your AS or A level Mathematics, you study some of the applications of mathematics (Mechanics and Statistics). You need to use modelling in these applications, and you will usually have to make simplifying assumptions about your work. You should always think about how the assumptions that you have made might affect your results. For example, if you were investigating the best angle to throw a ball so that it goes as far as possible, it would probably be realistic to assume that the effect of air resistance can be neglected. However, if you were looking at the time take for a parachute to descend, air resistance would be a very important factor, and to assume that it can be neglected would probably give you a very inaccurate result.