Section 1: Introduction to vectors

Notes and Examples

These notes contain subsections on

Vectors in magnitude-direction form or component form

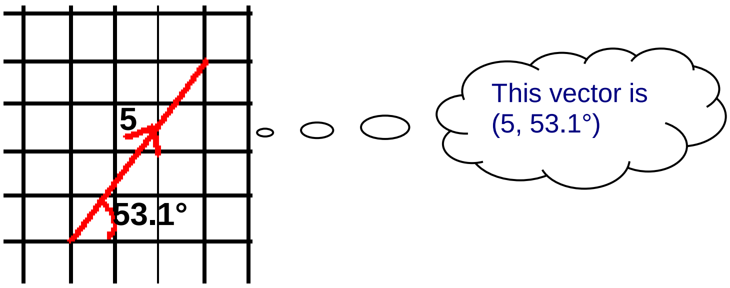
A **vector** quantity has both **magnitude** (size) and **direction**. A **scalar** quantity has magnitude only.

Vectors may be written in bold, **a**, or underlined, $\underline{a}$, or with an arrow above, $\vec{a}$.

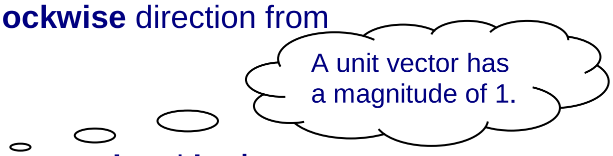
Two vectors are **equal** if they have the same magnitude and direction.

You need to be able to write down a vector in two different ways:

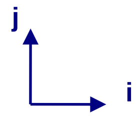
- Magnitude-direction form (r, θ)

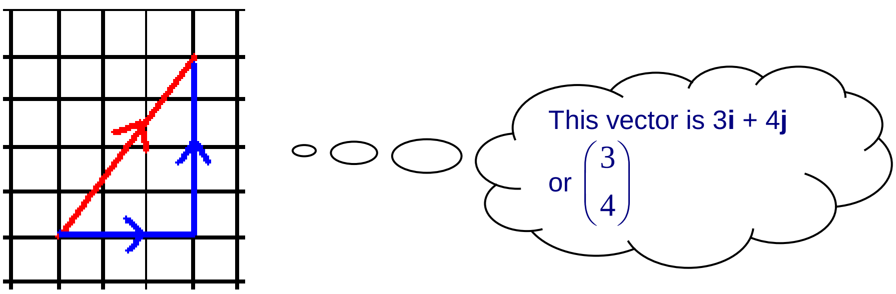


The angle, θ is measured in an **anticlockwise** direction from the **positive *x* axis**.



- Component form
  The vector is expressed using the unit vectors **i** and **j**.
  i is a unit vector in the x direction.
  j is a unit vector in the y direction.





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The magnitude of a vector given in component form is found using Pythagoras's theorem.
So the vector $\mathbf{c} = a\mathbf{i} + b\mathbf{j}$ has magnitude:

$$|\mathbf{c}| = \sqrt{a^2 + b^2}$$

The magnitude of a vector is sometimes called the **modulus**.

A vector given in magnitude-direction form can be written in component form using the rule:

$$\mathbf{a} = (r, \theta) \Rightarrow \mathbf{a} = \begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix} = r\cos\theta\mathbf{i} + r\sin\theta\mathbf{j}$$

The following two examples show you how to convert between the two forms.

Example 1
Write the vectors:
(i) (10, 70°) (ii) (5, 230°)
in component form.

Solution

(i) Using the formula $\mathbf{a} = (r, \theta) \Rightarrow \mathbf{a} = \begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix} = r\cos\theta\mathbf{i} + r\sin\theta\mathbf{j}$

$$\begin{aligned}(10, 70°) &= 10\cos 70°\mathbf{i} + 10\sin 70°\mathbf{j} \\ &= 3.42\mathbf{i} + 9.40\mathbf{j}\end{aligned}$$

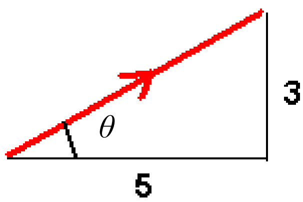
(ii)
$$\begin{aligned}(5, 230°) &= 5\cos 230°\mathbf{i} + 5\sin 230°\mathbf{j} \\ &= -3.21\mathbf{i} - 3.83\mathbf{j}\end{aligned}$$

Example 2
Write the vector:
(i) $5\mathbf{i} + 3\mathbf{j}$ (ii) $-2\mathbf{i} - 4\mathbf{j}$
in magnitude-direction form.

Solution

(i) The magnitude of the vector $5\mathbf{i} + 3\mathbf{j}$ is $\sqrt{5^2 + 3^2} = \sqrt{25 + 9} = \sqrt{34}$

Use a sketch to help you find the direction:



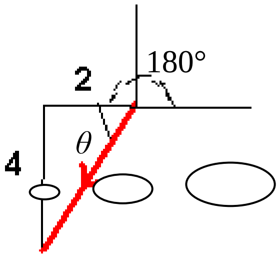
The angle θ gives the direction of the vector.

$\tan\theta = \frac{3}{5} \Rightarrow \theta = 31.0°$

So $5\mathbf{i} + 3\mathbf{j} = (\sqrt{34}, 31.0°)$

(ii) The magnitude of the vector $-2\mathbf{i} - 4\mathbf{j}$ is $\sqrt{(-2)^2 + (-4)^2} = \sqrt{4+16} = \sqrt{20}$

Use a sketch to help you find the direction:



Remember the direction is measured in an **anticlockwise** direction from the **positive *x* axis**.

The angle $\theta + 180°$ gives the direction of the vector.

$\tan\theta = \frac{4}{2} \Rightarrow \theta = 63.4°$ so the direction is $63.4° + 180° = 243.4°$

So $-2\mathbf{i} - 4\mathbf{j} = (\sqrt{20}, 243.4°)$

Multiplying a vector by a scalar

To multiply a vector by a scalar (number) simply multiply each of the components by the scalar.

Note:

- when the scalar is positive the direction of the vector remains the same but the length (or magnitude) of the vector increases by the same factor.
- when the scalar is negative the direction of the vector is reversed and again the length (or magnitude) of the vector increase.

Example 3

$\mathbf{a} = 2\mathbf{i} - 3\mathbf{j}$

(i) Find $4\mathbf{a}$

(ii) Find the value of $|\mathbf{a}|$

(iii) Write down the value of $|4\mathbf{a}|$

Solution

(i) $4\mathbf{a} = 4(2\mathbf{i} - 3\mathbf{j}) = 8\mathbf{i} - 12\mathbf{j}$

(ii) $|\mathbf{a}| = \sqrt{2^2 + (-3)^2} = \sqrt{4+9} = \sqrt{13}$

(iii) $|4\mathbf{a}| = 4|\mathbf{a}| = 4\sqrt{13}$

Example 4
$\mathbf{a} = 5\mathbf{i} - 7\mathbf{j}$
Find $-\mathbf{a}$.

This is the same as multiplying by -1.
Just reverse the signs!

Solution
$\mathbf{a} = 5\mathbf{i} - 7\mathbf{j}$
So $-\mathbf{a} = -5\mathbf{i} + 7\mathbf{j}$

Adding and subtracting vectors

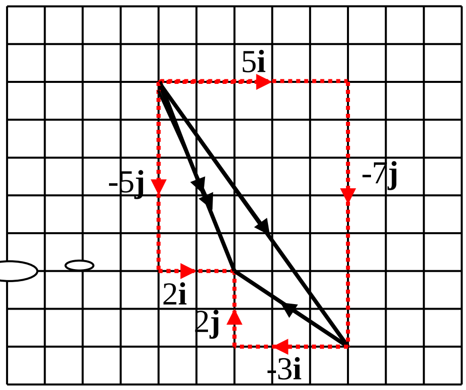
To add/subtract vectors simply multiply add/subtract the **i** components and then the **j** components.
Adding two or more vectors is called finding the **resultant**.

Example 5
(i) Find the resultant of $(5\mathbf{i} - 7\mathbf{j})$ and $(-3\mathbf{i} + 2\mathbf{j})$

(ii) Work out $\begin{pmatrix} 9 \\ -8 \end{pmatrix} - \begin{pmatrix} 5 \\ -3 \end{pmatrix}$

Solution
(i) To find the resultant you need to add the vectors.
$(5\mathbf{i} - 7\mathbf{j}) + (-3\mathbf{i} + 2\mathbf{j}) = 2\mathbf{i} - 5\mathbf{j}$
You can see this more clearly in this diagram:

The resultant is shown by a double headed arrow.



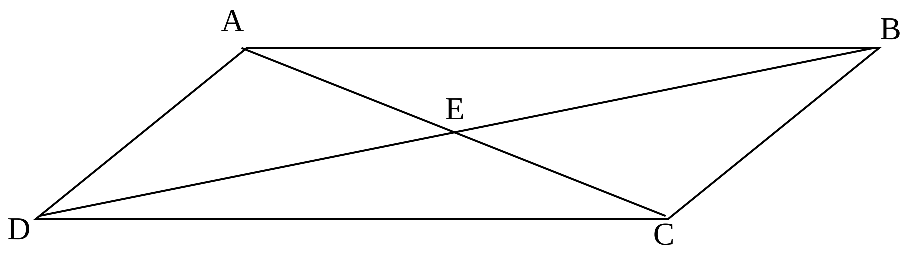
(ii) $\begin{pmatrix} 9 \\ -8 \end{pmatrix} - \begin{pmatrix} 5 \\ -3 \end{pmatrix} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$

The Explore resource ***Adding and subtracting vectors*** demonstrates the geometrical interpretation of vector addition and subtraction.

Equal vectors and position vectors

Two vectors are **equal** if they have the same magnitude and direction.
They do not have to be in the same place!

Example 6
The diagram shows a parallelogram ABCD.



$\overrightarrow{DA} = \mathbf{a}$
$\overrightarrow{AE} = \mathbf{b}$
$\overrightarrow{AB} = \mathbf{c}$

(a) Find in terms of **a**, **b** and **c** the vectors:
(i) $\overrightarrow{CB}$ (ii) $\overrightarrow{BC}$ (iii) $\overrightarrow{BD}$.

(b) Find two equivalent expressions for $\overrightarrow{AC}$.

Solution

(a) (i) $\overrightarrow{CB} = \overrightarrow{DA} = \mathbf{a}$
(ii) $\overrightarrow{BC} = -\overrightarrow{CB} = -\mathbf{a}$
(iii) $\overrightarrow{BD} = \overrightarrow{BA} + \overrightarrow{AD}$
$\overrightarrow{BD} = -\mathbf{c} - \mathbf{a}$

(b) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$
$\overrightarrow{AC} = \mathbf{c} - \mathbf{a}$
Also $\overrightarrow{AC} = 2\overrightarrow{AE} = 2\mathbf{b}$

A **position vector** is a vector which starts at the origin.
So if two position vectors are equal they will be in the same place!
For example the point A (5, -3) has the position vector $\overrightarrow{OA} = 5\mathbf{i} - 3\mathbf{j}$.

You need to know that

- $\overrightarrow{AO} = -\overrightarrow{OA}$
- $\overrightarrow{AB} = -\overrightarrow{OA} + \overrightarrow{OB}$
  So $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$
- The mid-point, M, has position vector:
  $\overrightarrow{OM} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$

You can see the reason for these results more clearly in this diagram:

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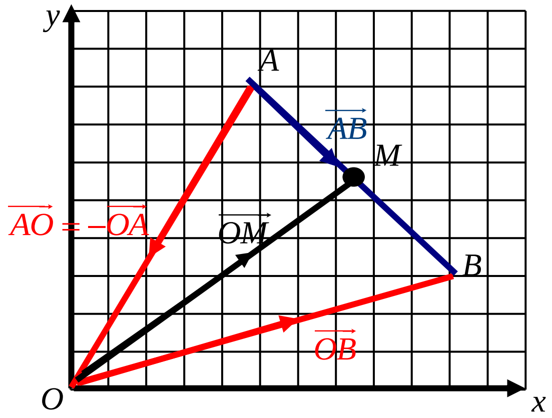


Example 7
The points A and B have coordinates (2, 4) and (5, -1) respectively.
(i) Write down the position vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$.
(ii) Find the vector $\overrightarrow{AB}$.
(iii) Find the position vector of the mid-point, M of AB.

Solution

(i) $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

$\overrightarrow{OB} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$

(ii) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 5 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$

(iii) $\overrightarrow{OM} = \overrightarrow{OA} + \tfrac{1}{2}\overrightarrow{AB}$

$$= \begin{pmatrix} 2 \\ 4 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 3 \\ -5 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} 1\tfrac{1}{2} \\ -2\tfrac{1}{2} \end{pmatrix} = \begin{pmatrix} 3\tfrac{1}{2} \\ 1\tfrac{1}{2} \end{pmatrix}$$

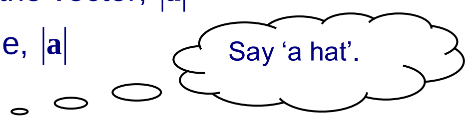
Unit vectors

A **unit vector** has a magnitude of 1.
i and **j** are examples of unit vectors.

You need to be able to find a unit vector which has the same direction as a given vector, **a.**
You do this by:

- Finding the magnitude of the vector, $|\mathbf{a}|$
- Dividing **a** by its magnitude, $|\mathbf{a}|$

The unit vector of **a** is written $\hat{\mathbf{a}}$.



Example 8

Find the unit vector in the direction of $\mathbf{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

Solution

$|\mathbf{a}| = \sqrt{2^2 + (-3)^2} = \sqrt{4+9} = \sqrt{13}$

$\hat{\mathbf{a}} = \frac{2}{\sqrt{13}}\mathbf{i} - \frac{3}{\sqrt{13}}\mathbf{j}$