Section 2: Trigonometric equations

Exercise level 3 (Extension)

1. [Make sure that your calculator is in degree mode throughout this question.]

   The rise and fall of the tide at Port A is modelled by

   $$y = 3\left(\cos\left(\frac{59x}{2}\right)^\circ + 1\right) \qquad 0 \leq x \leq 50$$

   where x is the number of hours since midnight on Sunday night, and y metres is the depth in the approach channel to the port.

   (i)   Sketch the graph of the tide.

   (ii) Find the times of high and low water on Tuesday, giving your answer to the nearest minute.

   (iii) It is planned to bring a vessel into Port A at some time on Tuesday, near midday. The vessel has a draught of 2 metres (i.e. it requires 2 metres of water to enter the port). Between what times on Tuesday must the vessel be brought into Port A?

   (iv) At a second port further along the coast, Port B, the times of high and low water occur 2 hours later than at Port A, and the depth of water in the approach channel to Port B varies from 1 to 5 metres. Adapt the formula given for Port A to give a similar formula for tides at Port B.

   (v)   At what times on Tuesday could the vessel be taken into Port B?

   (vi) Which of the ports is accessible for the longer period near Tuesday midday?

2. (i) Show that $x = \frac{1}{2}$ satisfies the equation
   $$32x^3 - 48x^2 + 22x - 3 = 0$$
   and hence factorise the equation fully.
   (ii) Hence find all solutions of
   $$32\cos^3 x - 48\cos^2 x + 22\cos x - 3 = 0 \quad \text{for } 0^\circ \leq x \leq 360^\circ$$