Section 1: Surds

Exercise level 3 (Extension)

1. When a simple pendulum swings gently, the period in seconds of the swing is given by the formula

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where L is the length of the pendulum in metres, and g is a constant which is the acceleration due to gravity. (Throughout this exercise, it is sufficiently accurate to use the approximation $g = 10 \text{ ms}^{-2}$.)

The diagram shows an experiment in which PQ is a simple pendulum, with a pendulum bob at Q, and the pendulum swinging L metres below the point P.
The end of a string of length 20 metres is fixed to the bob Q and passes over a peg at the fixed point P, around another peg at point B, with the other end fixed at A. Point A is 3 metres below point P, so that ABP is a right-angled triangle.
The length L can be changed by sliding the point B along the horizontal line AC. The length of AB is x metres as in the diagram.
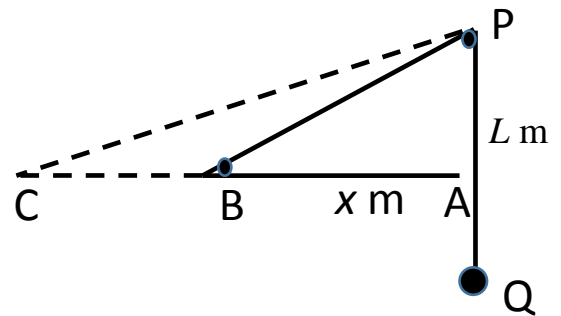
(i) Write down a formula for L, the length of the pendulum PQ, in terms of x.

(ii) Find the period of the pendulum when AB = 4 metres. Give your answer in surd form.

(iii) Write down a general formula for the period in terms of x.

(iv) Find the period of the pendulum when AB = 8 metres. Give your answer in surd form.

(v) If the period of the pendulum is T_1 when the length is L_1 and the period is T_2 when the length is L_2 then find a formula in terms of the lengths for the ratio of the time periods. Check that your answers to parts (ii) and (iv) above are consistent with your formula.