Section 2: The quadratic formula

Exercise level 3 (Extension)

1. When a stone is thrown upwards over the edge of a cliff, its height h metres above the point where it was thrown after t seconds is given by the formula
$$h = 20t - 5t^2$$
How many seconds after it is thrown does the stone pass the cliff edge on the way downwards? How long after it is thrown will the stone hit the sea which is 50 metres below the clifftop? (Give your answer to 3 significant figures.) How could you interpret the other solution of your quadratic equation?

2. (i)   Write the expression
$$x^2 + 8x + c$$
in completed square form.
    (ii) The equation
$$x^2 + 8x + c = 0$$
has real roots. Using the completed square format, find a condition in the form of an inequality for c.
    (iii) How must this condition on c be amended so that the equation above in part (ii) has real unequal roots?
    (iv) Sketch two graphs showing an example of two real equal roots in (ii) and two real unequal roots in (iii) above.
    (v)   By considering your sketch graphs, explain why there is no value of c which gives two real positive roots.

3. A rectangular car park has a perimeter of 184 metres, and the diagonal of the car park measures 68 metres.
    (i)   By labelling the length of the car park as x metres, formulate an equation and check that $x = 32$ satisfies the equation. Hence find the dimensions of the car park.
    (ii) Sketch the graph of the quadratic expression in part (i), and interpret each intersection with the x-axis in terms of the car park.