Edexcel AS Mathematics Problem solving

Section 2: Notation and proof

Exercise level 3 (Extension)

The idea of 'proof' and the correct use of the language of mathematics is fundamental to success in maths.

As you will have seen in your study, there are many ways of proving or disproving mathematical statements:

- Statements can be proved by various different methods including **exhaustion** (trying everything until you have used up all the possibilities) and logical **deduction** (often using algebra, based on previous knowledge)
- Statements are often disproved by a simple **counter-example** (often very difficult to find!).

As you work through these exercises, the important thing is to concentrate on the logic and the language – you do **not** need to 'learn' any of these proofs.

1. *(deduction)*
   Prove that for a number greater than or equal to 100 to be divisible by 4 it is necessary and sufficient that the number formed by its last two digits is a number divisible by 4.

2. *(deduction)*
   Prove that if $p, q \neq 0$,
   $x^2 - px + q = 0$ has two roots, one twice the other $\Leftrightarrow$ $9q - 2p^2 = 0$

3. *(counter example)*
   Disprove this statement
   "Every odd integer between 2 and 44 is either prime or the product of two primes"

4. *(exhaustion)*
   Find all the two digit numbers such that the sum of the first digit and the square of the second digit is the number itself. Are there ways of reducing the search so it is not necessary to check all 90 cases?

5. *(exhaustion)*
   (i) Show that $[10b+c]^2 = 10[10b+2c]b + c^2$
   (ii) Explain how this helps to prove that the squares of all positive integers can only end with certain final digits. Which digits can never be the final digit of a perfect square?