

## Section 2: Notation and proof

### Exercise level 2

The idea of ‘proof’ and the correct use of the language of mathematics is fundamental to success in maths.

As you will have seen in your study, there are many ways of proving or disproving mathematical statements:

- Statements can be proved by various different methods including **exhaustion** (trying everything until you have used up all the possibilities) and logical **deduction** (often using algebra, based on previous knowledge)
- Statements are often disproved by a simple **counter-example** (often very difficult to find!)

As you work through these exercises, the important thing is to concentrate on the logic and the language – you do **not** need to ‘learn’ any of these proofs.

1. (*deduction*)  
Prove that if  $n$  is an even number then  $n^2 + n$  is an even number
2. (*deduction*)  
Prove that
  - (i) the sum of five consecutive integers is a multiple of 5.
  - (ii) the mean of five consecutive integers is equal to the third number.
3. (*counter example*)  
The following statements are all false. Find a counter example that disproves them in each case
  - (i) the difference between any two square numbers is odd
  - (ii) all prime numbers are odd
  - (iii) for any positive integer  $n$ , the number  $n \times (n-1) \times (n-2) \times \dots \times 2 \times 1$  has  $2^{n-1}$  distinct factors
4. (*exhaustion*)  
Prove that if the name of a month has 5 or more characters, then a 4-letter word can be formed using those characters.