Section 1: Sketching curves

Notes and Examples

These notes contain subsections on

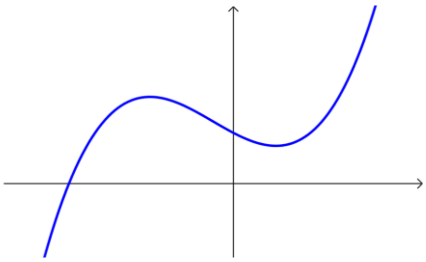
- Graphs of polynomial functions
- Reciprocal functions
- Points of intersection
- Proportional relationships

Graphs of polynomial functions

You have already met the graphs of quadratic functions $y = ax^2 + bx + c$, and polynomial functions such as $y = ax^3 + bx^2 + cx + d$.

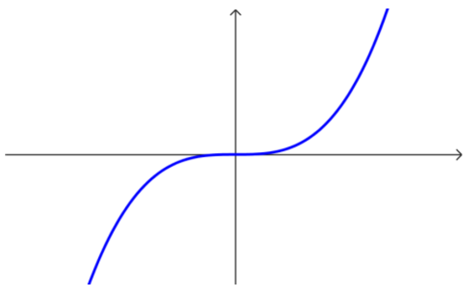
You have seen that a polynomial function of degree n has a graph which cuts the x-axis at most n times, and has at most $n-1$ turning points.

e.g. this is the graph of a cubic function, crossing the x-axis once, with 2 turning points.

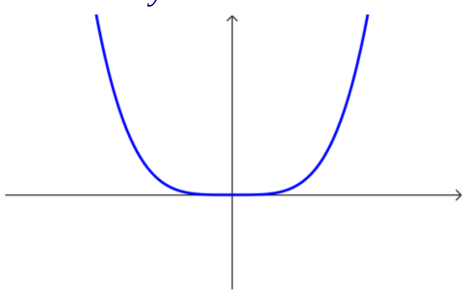


You also need to recognise the graphs of functions of the form $y = x^n$. These are also polynomials. They all cross the x-axis at the origin only.

This is the graph of $y = x^3$. Other functions of the form $y = x^n$ where n is odd have a similar shape. There are no turning points, but the graph is horizontal at the origin – this is called a point of inflection and you will learn more about points of inflection in A level mathematics.



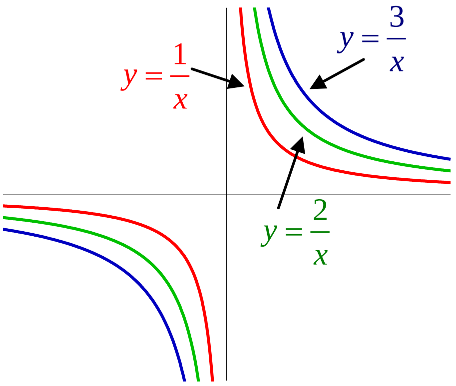
This is the graph of $y = x^4$. Other functions of the form $y = x^n$ where n is even have a similar shape. There is a minimum point at the origin, but it is flatter than the one for a quadratic graph.

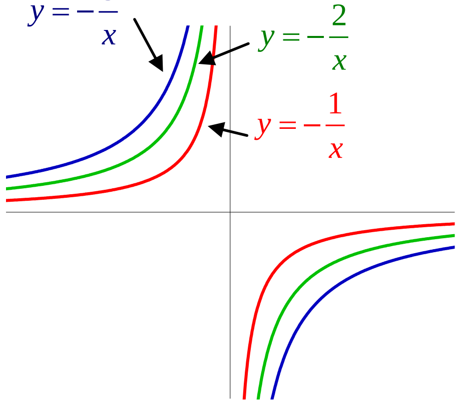


Edexcel AS Maths Graphs 1 Notes and Examples

Reciprocal graphs

You need to recognise and be able to sketch graphs of the form $y = \frac{k}{x}$, where k is a constant. The graphs have different shapes according to whether k is positive or negative.





All these curves have asymptotes $x = 0$ and $y = 0$ (the coordinate axes). In section 2 you will look at how these graphs can be translated, so that the asymptotes are different.

Points of intersection

At the points of intersection of two curves, the coordinates of the point of intersection satisfy the equations of both curves. By equating the two expressions for y, you can find an equation in x. The solutions of this equation give you the x-coordinates of the points of intersection.

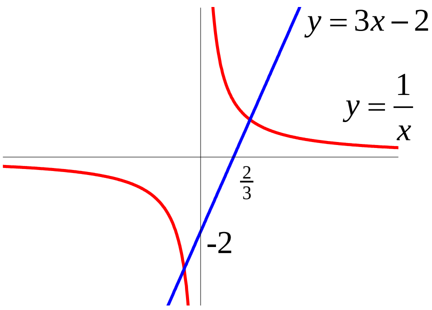
Example 1

(i) On the same axes sketch the curve $y = \frac{1}{x}$ and the line $y = 3x - 2$.

(ii) Find the points of intersection of the curve and the line.

Solution

(i) The line $y = 3x - 2$ passes through the points (0, -2) and ($\frac{2}{3}$, 0).



(ii) At the points of intersection, $3x - 2 = \dfrac{1}{x}$.

Multiply through by x: $3x^2 - 2x = 1$

$3x^2 - 2x - 1 = 0$

$(3x+1)(x-1) = 0$

$x = -\frac{1}{3}$ or $x = 1$

When $x = -\frac{1}{3}$, $y = \dfrac{1}{x} = -3$.

When $x = 1$, $y = \dfrac{1}{x} = 1$.

So the points of intersection are $\left(-\frac{1}{3}, -3\right)$ and (1, 1).

In the example above, you can see that the points of intersection are sensible by looking at the graph.

Proportional relationships

If y is **directly proportional** to x, then the relationship between x and y can be written as $y \propto x$ or as $y = kx$, where k is a constant. The graphs of proportional relationships are straight lines through the origin.

If y is **inversely proportional** to x, then the relationship between x and y can be written as $y \propto \dfrac{1}{x}$ or as $y = \dfrac{k}{x}$, where k is a constant. This means that as one quantity increases, the other decreases.

Other proportional relationships are also possible – for example if y is directly proportional to the square root of x, this would be written as $y \propto \sqrt{x}$ or $y = k\sqrt{x}$.

Example 2
A quantity P is inversely proportional to the square of another quantity d.
When $d = 2, P = 10$.
Find the value of P when $d = 18$.

Solution

$P = \dfrac{k}{d^2}$

When $d = 2, P = 10 \Rightarrow 10 = \dfrac{k}{2^2} \Rightarrow k = 10 \times 4 = 40$

$P = \dfrac{40}{d^2}$

When $d = 18$, $P = \dfrac{40}{18^2} = 0.123$ (3 s.f.)