Edexcel AS Mathematics Integration

Section 2: Finding the area under a curve

Notes and Examples

These notes contain subsections on:
- **Definite integration**
- **The definite integral as an area**

Definite integration

The definite integral from a to b of a function $f(x)$, which is written as

$\int_a^b f(x)dx$, is found as follows:

- Integrate $f(x)$ – suppose we call the integral $g(x)$
- Write the integral in square brackets, with the limits on the right hand side: $\left[g(x)\right]_a^b$
- Work out the value of $g(x)$ with $x = a$ and $x = b$, and subtract: $\left[g(x)\right]_a^b = g(b) - g(a)$.

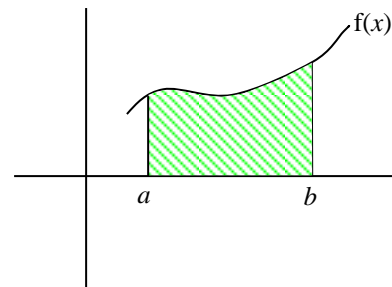**Example 1**

Evaluate $\int_1^2 (2x - x^2)dx$.

Solution

$$\int_1^2 \left(2x - x^2\right)dx = \left[x^2 - \tfrac{1}{3}x^3\right]_1^2$$
$$= \left(2^2 - \tfrac{1}{3} \times 2^3\right) - \left(1^2 - \tfrac{1}{3} \times 1^3\right)$$
$$= 4 - \tfrac{8}{3} - 1 + \tfrac{1}{3}$$
$$= \tfrac{2}{3}$$

Watch the signs here

The definite integral as an area

The definite integral $\int_a^b f(x)dx$ calculates the area between the curve $y = f(x)$ and the x-axis.

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If the curve is above the x-axis, so that the value of y is positive, the definite integral works out to be positive. However, if the curve is below the x-axis, so that y is negative, the integral works out to be negative.
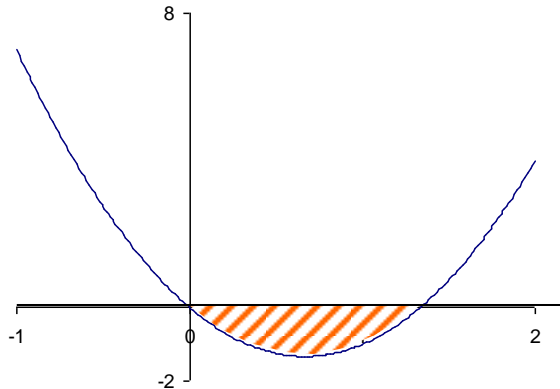
Example 2

Find the total area enclosed by the curve $y = 3x^2 - 4x$ and the x-axis.

Solution:

The curve crosses the x-axis when $3x^2 - 4x = 0$

$\Rightarrow \qquad x(3x - 4) = 0$

$\Rightarrow \qquad x = 0 \ \text{or} \ x = \frac{4}{3}$

$\text{Area} = \int_0^{\frac{4}{3}} (3x^2 - 4x)\,dx = \left[x^3 - 2x^2 \right]_0^{\frac{4}{3}}$

$= \left(\left(\tfrac{4}{3}\right)^3 - 2\left(\tfrac{4}{3}\right)^2 \right) - \left(0^3 - 2 \times 0^2 \right)$

$= -\frac{32}{27}$

This integral works out to be negative because the curve is below the x-axis.

The area is $\frac{32}{27}$ square units.

Notice that you should give your final answer as positive, since an area cannot be negative. However, remember that this only applies to definite integrals which are being used to find an area – if you are just asked to work out the value of a definite integral, then the answer may be positive or negative.