# **Edexcel AS Mathematics Integration**



## Section 1: Introduction to integration

These notes contain subsections on:

- Reversing differentiation
- The rule for integrating x<sup>n</sup>
- Indefinite integrals: formal notation
- Finding the arbitrary constant

### **Reversing differentiation**

Integration is the reverse of differentiation. If you are given an expression for  $\frac{dy}{dx}$ , and you want to find an expression for *y*, you need to use integration. This is sometimes called solving a differential equation.

Remember that when you integrate, you must always add an arbitrary constant (see the textbook for the explanation of this).

Example 1 shows how you can integrate a function by thinking about what function you would need to differentiate to obtain the given function.

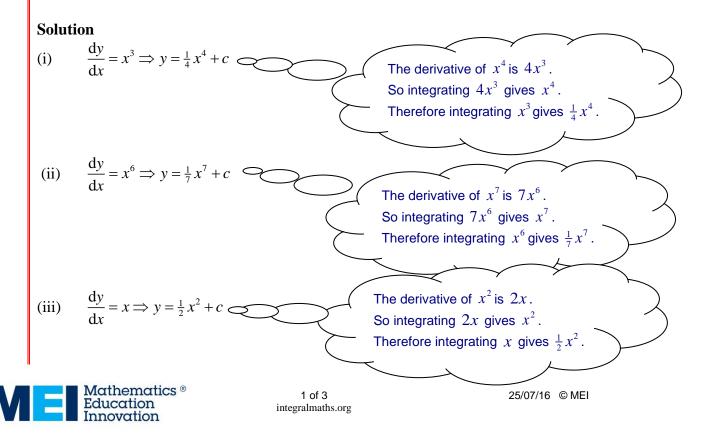


#### Example 1

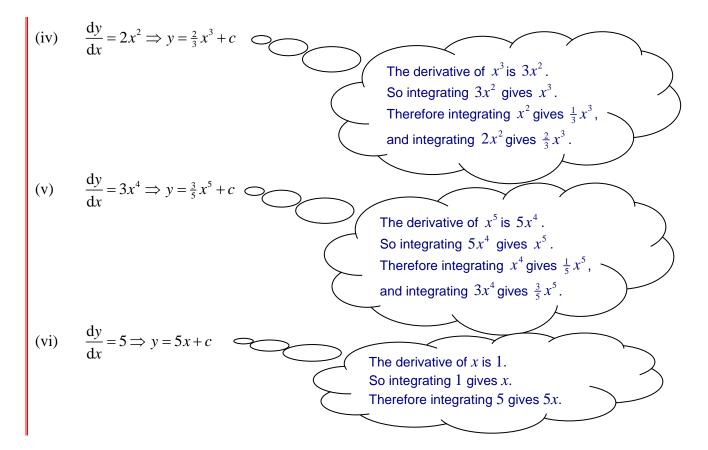
Find *y* as a function of *x* for each of the following.

(i) $\frac{dy}{dx} = x^3$ (iv) $\frac{dy}{dx} = 2x^2$	(ii) $\frac{\mathrm{d}y}{\mathrm{d}x} = x^6$	(iii) $\frac{\mathrm{d}y}{\mathrm{d}x} = x$
(iv) $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x^2$	(v) $\frac{dy}{dx} = 3x^4$	(vi) $\frac{dy}{dx} = 5$





## **Edexcel AS Maths Integration 1 Notes and Examples**



## The rule for integrating $x^n$

The method for integrating any polynomial function can be summed up as:

- Integrating  $x^n$ , where *n* is a positive integer, gives  $\frac{x^{n+1}}{n+1}$
- Integrating  $kx^n$ , where *n* is a positive integer and *k* is a constant, gives  $\frac{kx^{n+1}}{n+1}$
- You can integrate the sum of any number of such functions by simply integrating one term at a time.

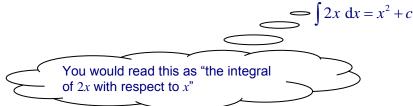
### Indefinite integration: formal notation

In Example 1 an expression for  $\frac{dy}{dx}$  was given and used to find an expression for *y*. So you would write:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2x \Longrightarrow y = x^2 + c \; .$$

# **Edexcel AS Maths Integration 1 Notes and Examples**

Using the formal notation, you would write this as:



The next example shows integration expressed using formal notation.

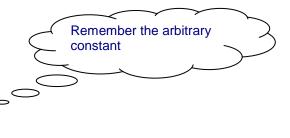


### **Example 2**

Solution

Integrate each of the following functions.

- $x^{3} + 3x + 2$ (i)
- $4x^2 5x 1$ (ii)
- (iii) (x+3)(x-2)





- Solution (i)  $\int (x^3 + 3x + 2) dx = \frac{1}{4}x^4 + \frac{3}{2}x^2 + 2x + c$ (ii)  $\int (4x^2 5x 1) dx = \frac{4}{3}x^3 \frac{5}{2}x^2 x + c$ (iii)  $\int (x+3)(x-2) dx = \int (x^2 + x 6) dx = \frac{1}{3}x^3 + \frac{1}{2}x^2 6x + c$

## Finding the arbitrary constant

If you are given additional information, you can find the value of the arbitrary constant by substituting the given information. This is sometimes called finding the particular solution of a differential equation. The next example shows how this is done.



#### **Example 3**

The gradient of a curve at any point (*x*, *y*) is given by  $\frac{dy}{dx} = x^2(2x + 1)$ .

The curve passes through the point (1, 5). Find the equation of the curve.



Solution: $\frac{dy}{dx} = x^2(2x+1) = 2x^3 + x^2$ Just as with differentiating, you need to expand the brackets first Integrating: $y = 2 \times \frac{1}{4}x^4 + \frac{1}{3}x^3 + c$ $= \frac{1}{2}x^4 + \frac{1}{3}x^3 + c$
When $x = 1$ , $y = 5$ $\Rightarrow 5 = \frac{1}{2} + \frac{1}{3} + c$ $\Rightarrow c = 5 - \frac{1}{2} - \frac{1}{3} = \frac{25}{6}$ Substitute the given values of x and y

So the equation of the curve is  $y = \frac{1}{2}x^4 + \frac{1}{3}x^3 + \frac{25}{6}$