Section 2: Inequalities

Notes and Examples

These notes contain subsections on
- Inequalities
- Linear inequalities
- Quadratic inequalities
- Dealing with fractions

Inequalities

Inequalities are similar to equations, but instead of an equals sign, =, they involve one of these signs:

$<$	less than
$>$	greater than
$\leq$	less than or equal to
$\geq$	greater than or equal to

This means that whereas the solution of an equation is a specific value, or two or more specific values, the solution of an inequality is a range of values.

Inequalities can be solved in a similar way to equations, but you do have to be very careful, as in some situations you need to reverse the inequality. This is shown in these examples.

Linear inequalities

A linear inequality involves only terms in x and constant terms.

Example 1

Solve the inequality $3x+1>x-5$

You can treat this just like a linear equation.

Solution

$$3x+1>x-5$$

Subtract x from each side

$$2x+1>-5$$

Subtract 1 from each side

$$2x>-6$$

Divide both sides by 2

$$x>-3$$

The next example involves a situation where you have to divide by a negative number. When you are solving an equation, multiplying or dividing by a negative number is not a problem. However, things are different with inequalities.

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The statement $-3 < 2$ is clearly true.

If you add something to each side, it is still true

$-3 + 1 < 2 + 1$
$-2 < 3$ ✓

If you subtract something from each side, it is still true

$-3 - 4 < 2 - 4$
$-7 < -2$ ✓

If you multiply or divide each side by a positive number, it is still true

$-3 \times 2 < 2 \times 2$
$-6 < 4$ ✓

However, if you multiply each side by a negative number then things go wrong!

$-3 \times -2 < 2 \times -2$
$6 < -4$ ✗

When you multiply or divide each side by a negative number, you must reverse the inequality.

The following example demonstrates this. Two solutions are given: in the first the inequality is reversed when dividing by a negative number, in the second this situation is avoided by a different approach.

Example 2
Solve the inequality $1 - x \geq 2x - 5$

Solution (1)

$1 - x \geq 2x - 5$ — Subtract $2x$ from each side

$1 - 3x \geq -5$ — Subtract 1 from each side

$-3x \geq -6$ — Divide both sides by -3, reversing the inequality.

$x \leq 2$

Solution (2)

$1 - x \geq 2x - 5$ — Add x to each side

$1 \geq 3x - 5$ — Add 5 to each side

$6 \geq 3x$ — Divide both sides by 3

$2 \geq x$ — Finish by writing the inequality the other way round.

$x \leq 2$

You can check that you have the sign the right way round by picking a number within the range of the solution, and checking that it satisfies the original inequality. In the above example, you could try $x = 1$. In the original inequality you get $0 \geq -3$, which is correct.

Quadratic inequalities

You can solve a quadratic inequality by factorising the quadratic expression, just as you do to solve a quadratic equation. This tells you the boundaries of the solutions. The easiest way to find the solution is then to sketch a graph.

Example 3

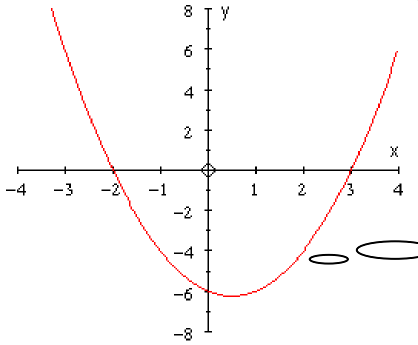
Solve the inequality $x^2 - x - 6 < 0$

Solution

$x^2 - x - 6 < 0$

$(x+2)(x-3) < 0$

This shows that the graph of $y = x^2 - x - 6$ cuts the x-axis at $x = -2$ and $x = 3$. Use this information to sketch the graph.



The solution to the inequality is the negative part of the graph. This is the part between –2 and 3.

The solution is $-2 < x < 3$

Notice that in Example 3, the solution is the set of values of x for which **both** $x > -2$ **and** $x < 3$. You can write this in set notation as

$\{x : x > -2) \cap \{x : x < 3\}$

The symbol $\cap$ is the 'intersection symbol' – so this means all the values of x that are in both sets

However, writing it in the form $-2 < x < 3$ is usually the clearest way to express the solution, as it shows that x lies between these two values.

Example 4

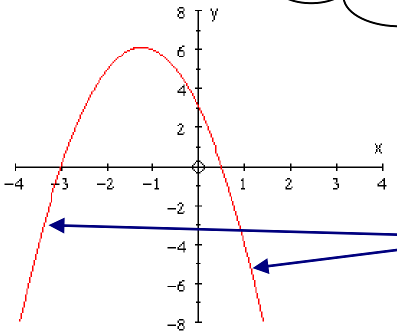
Solve the inequality $3 - 5x - 2x^2 \leq 0$

Solution

$3-5x-2x^2 \le 0$

$(3+x)(1-2x) \le 0$

This shows that the graph of $y=3-5x-2x^2$ cuts the x-axis at x = -3 and x = ½. You can now sketch the graph – note that as the term in x^2 is negative, the graph is inverted.



The solution to the inequality is the negative part of the graph. This is in fact two separate parts.

The solution is $x \le -3$ or $x \ge \frac{1}{2}$.

Notice the use of '**or**' in the solution. The value of x must be either less than or equal to 3, or greater than or equal to $\frac{1}{2}$ - it **cannot be both**, so the word 'and' must not be used. You cannot write this as a single inequality.

You can write the solution in set notation like this:

$\{x : x \le -3\} \cup \{x : x \ge \frac{1}{2}\}$

The symbol $\cup$ is the 'union' symbol and it means the combination of two sets, so it is all the values which are in either one set or the other.

Note: Example 4 involves a negative term in x^2. if you prefer to work with a positive x^2 term, you can change all the signs in the original inequality and reverse the inequality, giving $2x^2+5x-3 \ge 0$. The graph will then be the other way up, and you will take the positive part of the graph, so the solution will be the same.

For more practice, try the ***Inequalities skill pack***.

Dealing with fractions

If the unknown (such as x) is in the denominator of a fraction in an inequality, you must be careful. You should not multiply through by x to clear the fractions (as you would for an equation) because you don't know whether x is positive or negative.

You can get round the problem by multiplying through by x^2, as this is never negative. However, you must remember that you can only do this if $x \ne 0$, so $x = 0$ must be excluded from any solution set.

Example 5

Solve the inequality $\frac{2}{x} > 5$.

Solution

Multiply both sides by x^2: $2x > 5x^2 \qquad x \neq 0$

$5x^2 - 2x < 0$

$x(5x - 2) < 0$

$0 < x < \frac{2}{5}$