Edexcel AS Mathematics Surds and indices

Section 2: Indices

Notes and Examples

These notes contain subsections on
- **Multiplying expressions**
- **The rules of indices**
- **Negative indices**
- **Fractional indices**
- **More difficult examples**

Multiplying expressions

The example below illustrates multiplying expressions involving indices.

Example 1
Simplify the expression $2xy \times 3yz^2 \times 4x^2z$.

Solution
$$2xy \times 3yz^2 \times 4x^2z = 2 \times 3 \times 4 \times x \times x^2 \times y \times y \times z^2 \times z$$
$$= 24x^3y^2z^3$$

You may be happy to do this in your head, without writing out the intermediate line of working.

When you are multiplying expressions like the ones in Example 1, you are using one of the rules of indices.

The rules of indices

Three rules of indices are:

$$a^m \times a^n = a^{m+n}$$
$$a^m \div a^n = a^{m-n}$$
$$\left(a^m\right)^n = a^{mn}$$

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You can investigate these rules and see why they work by trying them out with simple cases, writing the sums out in full:

E.g., to demonstrate rule 3:

$$\left(2^3\right)^2 = \left(2\times2\times2\right)^2$$
$$= \left(2\times2\times2\right)\times\left(2\times2\times2\right)$$
$$= 2\times2\times2\times2\times2\times2$$
$$= 2^6$$
$$= 2^{3\times2}$$

Try some for yourself.

The number being raised to a power (a in this case) is called the **base**.
Note:
You can only apply these rules to numbers involving the same base.
So, for example, you cannot apply the rules of indices to $2^3 \times 3^5$.

Can you explain why?

Example 2
Simplify
(i) $2^4 \times 2^7$ (ii) $3^9 \div 3^4$ (iii) $(5^3)^6$ (iv) $2^3 \times 4^3$

Solution

(i) $2^4 \times 2^7 = 2^{4+7}$ using the first rule
$$= 2^{11}$$

(ii) $3^9 \div 3^4 = 3^{9-4}$ using the second rule
$$= 3^5$$

(iii) $(5^3)^6 = 5^{3\times6}$ using the third rule
$$= 5^{18}$$

(iv) $2^3 \times 4^3 = 2^3 \times (2^2)^3$ At first sight this looks as if it cannot be simplified, as the bases are different. However, 4 can be written as a power of 2.
$$= 2^3 \times 2^6$$
$$= 2^{3+6}$$
$$= 2^9$$

You can see and practise some similar examples using the *Laws of indices skill pack*.

Negative indices

There are two more rules, which follow from the three already introduced:

$$a^{-n} = \frac{1}{a^n}$$
$$a^0 = 1$$

> Again, it's worth experimenting with numbers to get a feel for how and why these rules work.
> e.g.
>
> $$2^{-1} = \frac{1}{2}$$
> $$\Rightarrow 2^1 \times 2^{-1} = 2 \times \frac{1}{2} = 1$$
>
> And from rule 3,
>
> $$2^1 \times 2^{-1} = 2^{1-1} = 2^0 = 1$$
>
> Try some for yourself.

Note that it might seem strange that $a^0 = 1$ for any value of a, but if this were not so, the other rules would be inconsistent. If you consider a graph of $y = a^x$, for different values of a, you will see that it is perfectly natural that $a^0 = 1$. Try this on your graphical calculator.

Example 3
Find, as fractions or whole numbers,
(i) 2^{-4} (ii) 5^{-2} (iii) 3^0

Solution
(i) $2^{-4} = \dfrac{1}{2^4} = \dfrac{1}{16}$

(ii) $5^{-2} = \dfrac{1}{5^2} = \dfrac{1}{25}$

(iii) $3^0 = 1$

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Fractional indices

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$$

> Although these are equivalent, it is usually easier to use the first form, working out the root first so that you are dealing with smaller numbers.

As before, try experimenting with numbers to get a feel for how an why these rules work.

Example 4

Find, as fractions or whole numbers,

(i) $8^{\frac{1}{3}}$ (ii) $9^{\frac{3}{2}}$ (iii) $25^{-\frac{1}{2}}$ (iv) $4^{-\frac{5}{2}}$

Solution

(i) $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

(ii) $9^{\frac{3}{2}} = (\sqrt{9})^3 = 3^3 = 27$

(iii) $25^{-\frac{1}{2}} = \dfrac{1}{25^{\frac{1}{2}}} = \dfrac{1}{\sqrt{25}} = \dfrac{1}{5}$

(iv) $4^{-\frac{5}{2}} = \dfrac{1}{4^{\frac{5}{2}}} = \dfrac{1}{(4^{\frac{1}{2}})^5} = \dfrac{1}{(\sqrt{4})^5} = \dfrac{1}{2^5} = \dfrac{1}{32}$

You can see and practise more examples like the ones above using the *Negative, fractional and zero indices skill pack*. You can also look at the *Indices video*.

More difficult examples

The next example shows how you can sometimes simplify quite complicated looking expressions involving different bases by splitting them up into their factors.

Example 5

Simplify $6^{5/2} \times \dfrac{1}{\sqrt{12}} \times 18^{-3/2}$

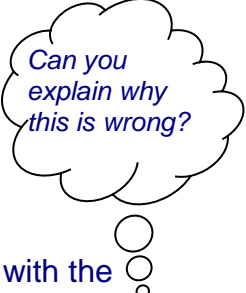# Edexcel AS Maths Surds and indices 2 Notes & Examples

Solution

$6^{5/2} = (2 \times 3)^{5/2} = 2^{5/2} \times 3^{5/2}$

$\dfrac{1}{\sqrt{12}} = \dfrac{1}{\sqrt{4}} \times \dfrac{1}{\sqrt{3}} = \dfrac{1}{2} \times \dfrac{1}{3^{1/2}} = 2^{-1} \times 3^{-1/2}$

$18^{-3/2} = 9^{-3/2} \times 2^{-3/2} = (9^{1/2})^{-3} \times 2^{-3/2} = 3^{-3} \times 2^{-3/2}$

$6^{5/2} \times \dfrac{1}{\sqrt{12}} \times 18^{-3/2} = 2^{5/2} \times 3^{5/2} \times 2^{-1} \times 3^{-1/2} \times 3^{-3} \times 2^{-3/2}$

$\qquad\qquad\qquad = (2^{5/2} \times 2^{-1} \times 2^{-3/2}) \times (3^{5/2} \times 3^{-1/2} \times 3^{-3})$

$\qquad\qquad\qquad = 2^{0} \times 3^{-1}$

$\qquad\qquad\qquad = 1 \times 3^{-1}$

$\qquad\qquad\qquad = \dfrac{1}{3}$

Can you explain why this is wrong?

A common mistake when dealing with indices is to try to add terms with the same base but a different index, such as $2^3 + 2^5$, by adding the indices. This is wrong, but you can sometimes simplify expressions like this by taking out a common factor. This is shown in the example below.

Example 6

Simplify $2^{5/2} + 2^{1/2}$.

Solution

$2^{5/2} + 2^{1/2} = 2^2 \times 2^{1/2} + 2^{1/2}$

$\qquad\qquad = 4 \times 2^{1/2} + 2^{1/2}$

$\qquad\qquad = 5 \times 2^{1/2}$

$\qquad\qquad = 5\sqrt{2}$