## Section 2: Indices

### Exercise level 3 (Extension)

1. Kepler's third law of planetary motion states that the square of the period $T$ of an orbit is proportional to the cube of the radius $r$. (The exact law is slightly differently stated, but this version is correct for circular orbits.)

   (i) Write down a formula for this law, in terms of $T$, $r$, and $k$, the constant of proportionality.

   (ii) Using your formula, express $T$ in terms of $r$ and $k$, and also express $r$ in terms of $T$ and $k$.

   (iii) In the rest of this question, we measure $T$ in days and $r$ in kilometres. The Moon orbits the Earth in 27.3 days, on a path which is nearly circular, with a radius of 382300 kilometres. Find the value of $k$.

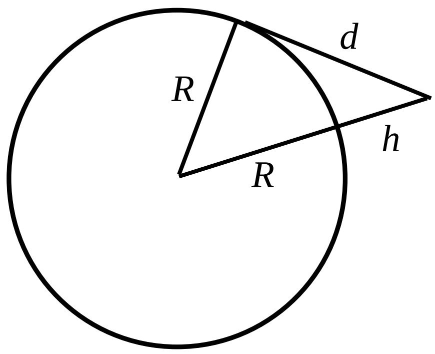
   (iv) A geostationary satellite is one which orbits in exactly one day. What must be the radius of its orbit?

   (v) A circumpolar research satellite is in a circular orbit with a radius of 110000 kilometres. What is the period of its orbit?

   (vi) A permanent Space Station orbits the Earth in a circular orbit taking just one and a half hours. How high above the ground does it orbit? (The Earth's radius is 6371 km.)

2. My old copy of "The Young Person's Guide to the Coast" states that for an observer looking out to sea from a height of $h$ metres above sea level, the horizon at sea is at a distance of approximately $3.57\sqrt{h}$ kilometres.

   (i) In the diagram on the right, $R$ is the radius of the Earth, $h$ is the height above sea level of the observer, and $d$ is the distance to the horizon. Show that this leads to the formula
   $$d = \sqrt{h}\sqrt{2R+h}.$$

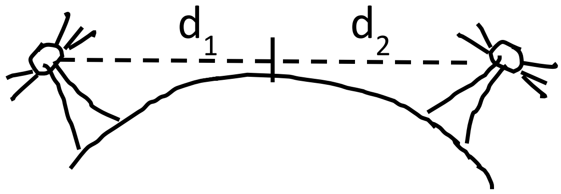



   (ii) Explain how, if $h$ is very small in comparison to the radius of the Earth $R$, then this formula can be approximated by $d = \sqrt{2Rh}$.
   The radius of the Earth is 6371 kilometres. Show that if $h$ is measured in metres, and $d$ in kilometres, then this formula gives $d \approx 3.57\sqrt{h}$

   (iii) Find the approximate distance to the horizon for
   (a) an observer with eye-level 2 m, standing on the beach.
   (b) the view from the edge of a 100 m high cliff.
   (c) the view from the top of a 1000 m mountain on the coast.

(d) the view from the cabin of an aircraft at altitude 10000 m.

(iv) The "Young Person's Guide" tells the tale of two Cornish lighthouses at Penvose and Tredeen, which are built many kilometres apart, but from which, by coincidence, each can see the other's light at low tide, but each is obscured from each other by the curve of the Earth at high water. The Penvose light is 59 m above the high tide, and the Tredeen light is 62 m above the high tide. At low water, each is a further 6 m above the sea. What is the greatest and least distance apart for the two lighthouses for this tidal anomaly to be true?




3. The 'fast chiller' in an industrial kitchen can cool hot food so that the change in temperature $\theta^{o}$ in a minute is given by the formula

$$k(\theta+10)^{\frac{5}{3}}$$

(i) When the chef places food at 30$^{o}$ in the chiller, it cools by 2$^{o}$ during the first minute. Find the approximate value of $k$.

(ii) A food scientist converts the formula above for the rate into a new formula linking time, $t$ minutes, and temperature $\theta$ $^{o}$, which is

$$t = C - \frac{3}{2k}\left(\theta+10\right)^{-\frac{2}{3}}$$

where $C$ is a constant depending on the initial temperature of the food. Initially, when $t$ = 0, the food has temperature 30 $^{o}$. Use the formula to find the value of the constant $C$, and determine how long it will take the food to be chilled to freezing point at zero degrees.