Section 3: Modelling curves

Exercise level 3 (Extension)

1. An experimenter finds the following data:

x	-4	-3	-2	-1	0	1	2
y	-1.99	-1.96	-1.88	-1.6	-0.89	1.41	8.57

She expects the data to follow a rule $y = C + ak^x$.

(i) Sketch the graph of y against x, and suggest a suitable value for C.

(ii) Explain how by a suitable change in the formula, the usual 'logarithm method' is still appropriate to find a and k.

(iii) Draw a suitable graph, and find the rule approximately.

2. Many astronomers over centuries noted an apparent 'rule' for the orbital distances of the planets from the Sun. This became known as Bode's Law.
At the time that the 'rule' first appeared, the orbital distance of Mercury was still uncertain, and Neptune (and Pluto) were still undiscovered, though the dwarf planet Ceres (in the asteroid belt) was recognised as a planet.
Numbering the planets from 1 (Venus) to 7 (Uranus) and taking the Earth's distance from the Sun as 1 (now known as an 'astronomical unit' (AU)), the planets were categorised as:

	Venus	Earth	Mars	Ceres	Jupiter	Saturn	Uranus
number n	1	2	3	4	5	6	7
radius R	0.7	1	1.6	2.8	5.2	10	19.6

Bode's Law was sometimes written as $R = \frac{1}{10}\left(ab^n + 4\right)$

(i) Draw a suitable graph, and use it to determine Bode's Law.

(ii) Use Bode's Law to make 'predictions' for the orbital radii of Mercury (n = 0), Neptune (n = 8), and Pluto (n = 9).

(iii) The following table shows approximately the real known distances, together with the Bode's Law predictions.

Planet	n	Bode's Law (AU)	True (AU)
Mercury	0		0.39
Venus	1	0.7	0.72
Earth	2	1	1
Mars	4	1.6	1.52
Ceres	8	2.8	2.77
Jupiter	16	5.2	5.2
Saturn	32	10	9.54
Uranus	64	19.6	19.2
Neptune	128		30.06
Pluto	256		39.44

Add your calculations from (ii) to the table, and comment on the usefulness of Bode's Law.