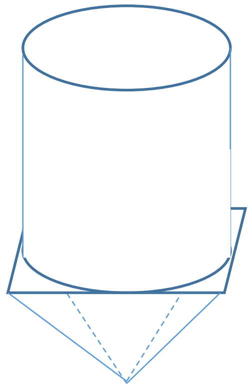
Section 4: More about differentiation

Exercise level 3 (Extension)

1. In a right-angled triangle, the two sides adjacent to the right angle have a fixed total length of k. Show that the largest possible area occurs when the triangle is isosceles, and find its area.

2. A grain hopper, as shown in the diagram on the right, consists of a cylinder of radius $\frac{1}{2}h$ and height h, fixed above an inverted square-based pyramid forming a 'funnel' where the square base has side equal to the diameter of the cylinder, and depth d, with all dimensions in metres.
In order that the grain flows out freely from the hopper, the total height of the structure must equal 15 metres less the square of the edge of the base of the pyramid.
(i) Find an expression for V m³, the volume of the hopper and funnel together.
(ii) Find the dimensions to give the maximum volume of the hopper and funnel together.



3. One statement of differentiation from first principles is

$$y = \mathrm{f}(x)$$

$$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Find from first principles, $\mathrm{f}'(x)$ if
(i) $\mathrm{f}(x) = x^2$
(ii) $\mathrm{f}(x) = \dfrac{1}{x^2}$,
carefully explaining your reasoning.

4. In a chemical process, a holding tank fills with liquid, while simultaneously the liquid flows out again. Initially (at time $t = 0$, measured in minutes), the tank is empty.
The rate at which the liquid flows into the tank is modelled by

$$y = -t^2 + 10t \quad \text{where } y \text{ is measured in litres/minute}$$

while the rate at which the liquid flows out of the tank is modelled by

$$y = -\tfrac{1}{10}t^3 + \tfrac{6}{5}t^2$$

For the first 10 minutes of operation, find:
(i) when the tank is filling at its quickest rate, and find the rate
(ii) at what time the tank stops filling and begins to empty
(iii) how quickly the tank is filling or emptying at the end of the 10 minute operation.