Section 4: More about differentiation

Exercise level 2

1. A variable rectangle has a constant perimeter of 20 cm. Find the lengths of the sides when the area is a maximum.

2. A square of side x cm is cut from the corners of a piece of card 15 cm by 24 cm. The card is then folded to form an open box.
   (i) Show that the volume of the box is $4x^3 - 78x^2 + 360x$ cm^3.
   (ii) Find a value for x that will make the volume a maximum.

3. A cylinder is cut from a solid sphere of radius 3cm. The height of the cylinder is $2h$.
   (i) Find the radius of the cylinder in terms of h.
   (ii) Show that the volume of the cylinder is $V = 2\pi h(9 - h^2)$.
   (iii) Find the maximum volume of the cylinder as h varies.

4. A cylindrical oil storage tank of radius r and height h is made so that the sum of its radius and its height is 24 m.
   Find the maximum volume of the storage tank.

5. A cylindrical can with height h metres and radius r metres has a capacity of 2 litres.
   (i) Find an expression for h in terms of r.
   (ii) Hence find an expression for the surface area of the can in terms of r only.
   (iii)Find the value of r which minimises the surface area of the can.

6. Find the gradient of the chord joining the point with x-coordinate -2 to the point with x-coordinate -2 + h on the curve $y = x^3 + 2x^2$.

7. The point P on the curve $y = 1 - x - x^3$ has x-coordinate -1.
   (i) Find the gradient of the chord joining P to the point on the curve with x-coordinate -1 + h.
   (ii) Hence find the gradient of the tangent to the curve at P.

8. Differentiate from first principles to find the derivative of each of the functions below.
   (i) $f(x) = 2x^2 - 3x + 1$
   (ii) $f(x) = x^3 - 2x^2 + 3$