Section 4: More about differentiation

Exercise level 1

1. Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ for each of the following.

   (i) $y = x^3 - 3x^2 + 4x - 1$

   (ii) $y = \frac{1}{x} - \frac{2}{x^2}$

   (iii) $y = 2\sqrt{x}$

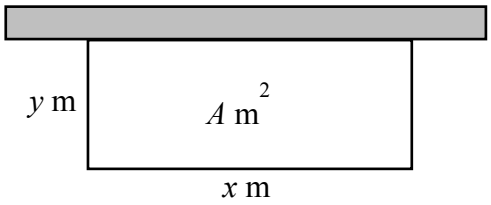
2. A curve has equation $y = x^3 - 3x^2 + 6$.

   (i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$.

   (ii) Find the coordinates of any turning points and use $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ to determine the nature of the turning points.

   (iii) Sketch the curve.

3. A farmer has 100 m of fence available, with which he intends to build a pen for his sheep. He intends to create a rectangular pen against a permanent stone wall, as in the diagram.



   (i) Show that $A = \frac{1}{2}x(100 - x)$.

   (ii) Find $\frac{\mathrm{d}A}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2A}{\mathrm{d}x^2}$.

   (iii) Find the value of x that makes the area as large as possible, and explain how you know that this is a maximum.

4. Find the gradient of the chord joining the point with x-coordinate 1 to the point with x-coordinate $1 + h$ on the curve $y = x^2 - 3x + 1$.

5. The point P on the curve $y = 2x^2 - x - 1$ has x-coordinate 1.

   (i) Find the gradient of the chord joining P to the point on the curve with x-coordinate $1 + h$.

   (ii) Hence find the gradient of the tangent to the curve at P.