Edexcel AS Mathematics Differentiation
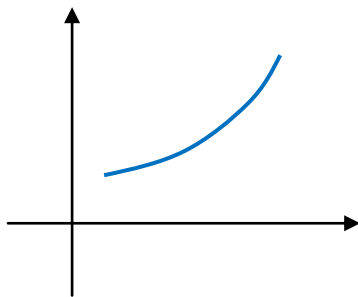
Section 2: Maximum and minimum points

These notes contain sub-sections on:
- **Increasing and decreasing functions**
- **Turning points**
- **Sketching the graph of a derivative**
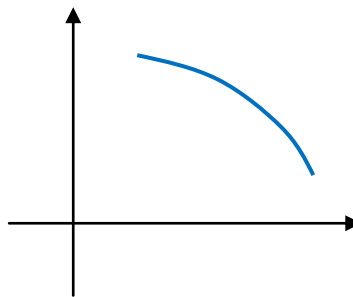
Increasing and decreasing functions

When the gradient $\dfrac{dy}{dx}$ of a graph is positive, the value of y is increasing.

Similarly, when the gradient is negative, the value of y is decreasing.

$$\frac{dy}{dx} > 0$$
Increasing function

$$\frac{dy}{dx} < 0$$
Decreasing function

Example 1

Find the range of values of x for which $y = x^3 - 3x^2 - 9x + 4$ is increasing.

Solution

$y = x^3 - 3x^2 - 9x + 4$

$\dfrac{dy}{dx} = 3x^2 - 6x - 9$
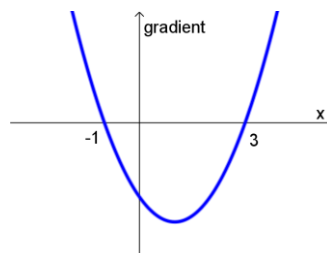
The function is increasing if $\dfrac{dy}{dx} > 0$

$\Rightarrow 3x^2 - 6x - 9 > 0$

$\Rightarrow x^2 - 2x - 3 > 0$

$\Rightarrow (x-3)(x+1) > 0$

$\Rightarrow x < -1 \text{ or } x > 3$

So the function is increasing for $x < -1$ and for $x > 3$

# Edexcel AS Maths Differentiation 2 Notes and Examples

Turning points

Points on a curve where the tangent is horizontal are called *stationary points*, or *turning points.*

At these points, the gradient of the curve is zero, so $\dfrac{dy}{dx} = 0$.

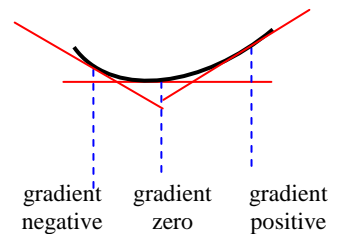You will be looking at two types of stationary point:

Local **maximum**

The gradient is positive to the left, zero at the point, and negative to the right.

gradient positive   gradient zero   gradient negative
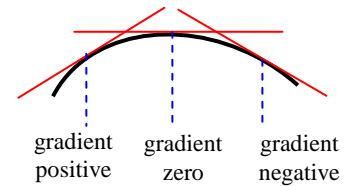
Local **minimum**

The gradient is negative to the left, zero at the point, and positive to the right.

gradient negative   gradient zero   gradient positive

To distinguish between these, you can test the value of the derivative either side of the stationary point, to see whether the gradient is positive or negative.

Example 2

Find the stationary points on the curve $y = x^3 - 3x^2 + 1$, investigate their nature, and sketch the curve.

Solution

$y = x^3 - 3x^2 + 1$

$\dfrac{dy}{dx} = 3x^2 - 6x$

> **Step 1:** Differentiate the function.

$3x^2 - 6x = 0$

$3x(x - 2) = 0$

$x = 0$ or $x = 2$

> **Step 2:** Solve $\dfrac{dy}{dx} = 0$

When $x = 0$, $y = 0^3 - 3 \times 0^2 + 1 = 1$

When $x = 2$, $y = 2^3 - 3 \times 2^2 + 1 = 8 - 12 + 1 = -3$.

> **Step 3:** Calculate the y–coordinates for these values of x (called the stationary values).

So the stationary points are (0, 1) and (2, −3).

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Edexcel AS Maths Differentiation 2 Notes and Examples

Step 4: Use a table to investigate the sign of $\dfrac{dy}{dx}$

for values of x either side of the stationary values

x	-1	0	1	2	2
$\dfrac{dy}{dx}$	9 +ve	0	-3 -ve	0	9 +ve

So (0, 1) is a local maximum and (2, -3) is a local minimum
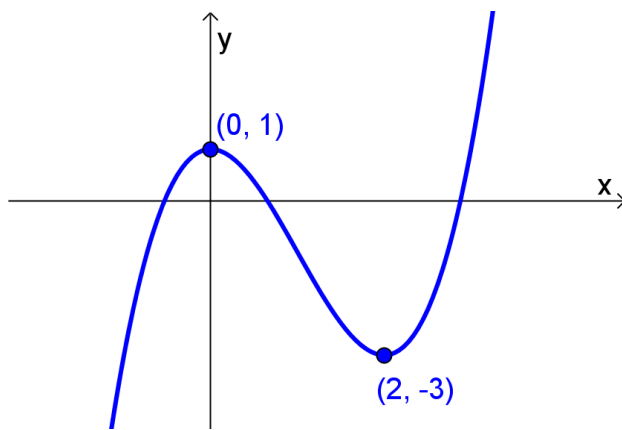
Step 5: Sketch the curve.

You can investigate stationary points on quadratic and cubic curves using the Explore resource ***Stationary points***. Also try the ***Stationary points walkthrough***.

## Sketching the graph of a derivative

If you have the graph of a function $y = f(x)$, you can sketch the graph of the corresponding gradient function, $y = f'(x)$, by thinking about what is happening at different points on the graph.

- Where there is a turning point, the gradient of $y = f(x)$ is zero so the gradient graph $y = f'(x)$ crosses the x-axis
- Where the graph is increasing, the gradient of $y = f(x)$ is positive so the gradient graph will be above the x-axis
- Where the graph is decreasing, the gradient of $y = f(x)$ is negative so the gradient graph will be below the x-axis

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Example 3

Sketch the graph of the gradient function of the curve shown below.

$y = \mathrm{f}(x)$
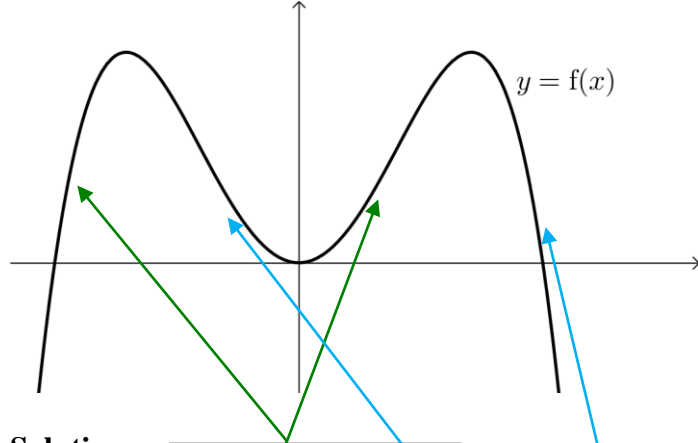
Solution

Here the gradient is positive

$y = \mathrm{f}\,'(x)$
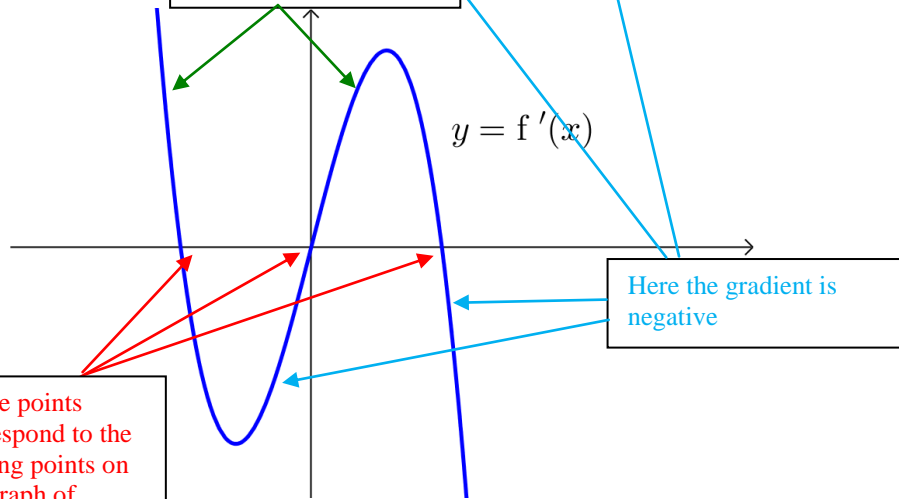
Here the gradient is negative

These points correspond to the turning points on the graph of $y = \mathrm{f}'(x)$, where the gradient is zero