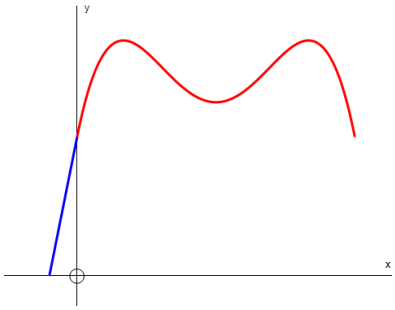
Section 2: Maximum and minimum points

Exercise level 3 (Extension)

1. Part of a leisure park ride is modelled by

$$y = \tfrac{1}{3}\left(-\tfrac{1}{4}x^4 + 3x^3 - \tfrac{23}{2}x^2 + 15x + k\right)$$

between $x = 0$ and $x = 6$, where each unit represents 10 metres on the full-sized ride. [After $x = 6$, the ride disappears into a tunnel, and we shall not consider this part!]



(i) The part modelled above starts 30 metres above the horizontal ground through the origin. Find the value of k.

(ii) The ride is approached when $x < 0$ by a straight ramp of constant gradient up which the ride 'cars' are hauled. The ramp meets the curve at $x = 0$ smoothly, with the same gradient. Find the equation of the ramp in the model.

(iii) The ride is designed so that the first 'peak' occurs 10 metres horizontally after the start of the curve. Find the positions of the 'peaks' and 'dips' on the real ride.

(iv) Find the places on the ride where the track is steepest.

2. A ski jump ramp is modelled by the equation

$$y = 0.03x^3 - 0.07x^2 - 1.2x + 6$$

between $x = 0$ and $x = 5$, where each unit of x and y represents 10 metres, measured horizontally or vertically from an origin located at the base of the ski jump ramp.

A ski jumper skis down the ramp, and takes off from the end of the ramp. His subsequent jump is modelled by the equation

$$y = -0.2x^2 + 2.5x - 5.5$$

when $x > 5$.

(i) What is the height of the start of the ramp above the ground?

(ii) How far below the top of the ramp is the jumper at the point where he leaves the ski jump ramp?

(iii) What is the gradient of the ramp at the take-off point?

(iv) What is the initial gradient at the take-off point of the equation which models the subsequent jump?

(v) Suggest a reason why the answers to parts (iii) and (iv) are close, but not quite equal.

(vi) The jumper lands on sloping ground, where the ground at his landing point is modelled by the straight line $y = -x + 9.8$. By finding and solving a

quadratic equation, find the possible x-coordinates of the landing point. Explain why only one of these is valid.

(vii) Find the coordinates of the jumper's landing point, and the gradients of the 'flightpath' and the landing zone.

(viii) Write down the horizontal distance travelled in the jump from the take-off point to the landing point.

(ix) How far below the top of the ramp is the landing point?

(x) Why must the hill be constructed so that the two gradients you found in part (vii) are nearly equal?