Section 3: Extending the rule

Exercise level 2

1. Differentiate the following expressions with respect to x:

   (i) $y = x^{-\frac{1}{2}}$

   (ii) $y = -\dfrac{1}{x^4}$

   (iii) $y = (x-2)^2\sqrt{x}$

   (iv) $y = \dfrac{(x+1)^2}{\sqrt{x}}$

2. Find the coordinates of the point P where the graph of $y = x^2 + \dfrac{1}{x}$ crosses the x-axis, and hence find the equations of the tangent and normal through the point P.

3. Find any stationary points on the following curves and determine their nature.

   (i) $y = x - \dfrac{4}{x^2}$

   (ii) $y = \sqrt{x} + \dfrac{1}{\sqrt{x}}$

4. The point P with x-coordinate 1 lies on the curve $y = \dfrac{2}{x}$.

   Q is the point where the normal at P meets the curve again.
   R is the point where the tangents at P and Q meet.
   Find the area of triangle PQR. Give your answer in exact form.