

Section 1: Introduction to differentiation



Exercise level 3 (Extension)

1. (i) Sketch the graph of $y = -x^2 + 5x - 6$, and find P and Q where it crosses the x -axis.
 (ii) Find the gradients of the tangents at P and Q, and hence find the coordinates of the points A and B where the tangents and normals at P and Q intersect.
 (iii) Find the area of quadrilateral PAQB.

2. (i) Sketch the curve $y = \frac{1}{8}x^2$, and find the equation of the line through F (0, 2) with gradient $-\frac{1}{2}$. Add this line to your sketch.
 (ii) Find the x -coordinates of the points P and Q where the straight line and the curve in part (i) intersect.
 (iii) Find the gradients of the tangents to the curve at P and Q.
 (iv) Show that the tangents at P and Q are perpendicular.
 (v) Find the x -coordinates of points R and S, where the line $y = ax + 2$ intersects the curve.
 (vi) Find the gradients of the tangents to the curve at R and S.
 (vii) Show that the tangents at R and S are perpendicular.
 (viii) By considering your results, make a general statement about lines through the point F, and tangents to the curve.

3. (i) Find the gradients of each of the following graphs, at the points where they cross:
 $y = x^2 - 2$ and $y = -x + 4$
 (ii) Find the acute angle between the two graphs at the crossing point with the larger x -coordinate.

4. The ‘fringe’ at the top of a wooden fence is cut by a computer-controlled machine, as in the diagram. The curved sections of the fringe are given by the three quadratic equations

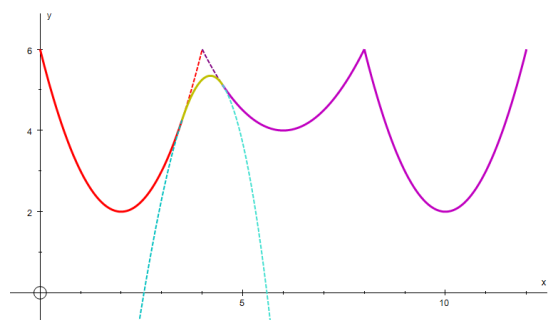
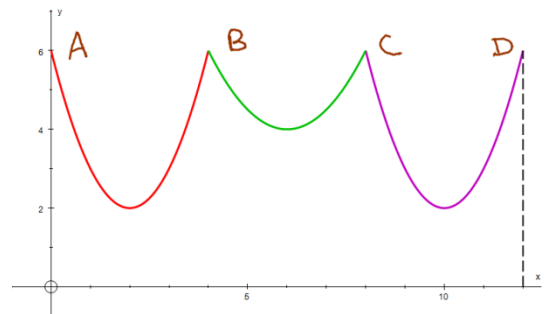
$$y = (x - 2)^2 + 2$$

$$y = \frac{1}{2}x^2 - 6x + 22$$

$$y = (x - 10)^2 + 2$$

between $x = 0$ and $x = 12$.

- (i) Find the angles at the vertices A, B, C, and D.
- (ii) After customer complaints about the sharp points, the manufacturer decides to ‘smooth’ the points. Considering point B, he decides to add a cubic curve from the point where $x = 3.5$ to the point where $x = 4.5$, as in the



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diagram. He can make the gradients fit at the left hand point. He chooses the curve

$$y = -0.25x^3 + 0.75x^2 + 6.938x - 18.5$$

Check that the new curve which the computer-controlled machine will cut meets the requirement to fit the old curve fairly accurately.

- (iii) Find the gradients of the old curves and that of the new curve at $x = 3.5$ and $x = 4.5$, and comment on the results.