Section 1: Introduction to differentiation

Exercise level 2

1. Given that $y = x^3 + 2x^2$, find $\frac{dy}{dx}$. Hence find the x–coordinates of the two points on the curve where the gradient is 4.

2. (i) Show that the point (1, 2) lies on both the curves $y = 2x^3$ and $y = 3x^2 - 1$.
   (ii) Show that the curves have the same gradient at this point.
   (iii) What do these results this tell you about the two curves?

3. The displacement s metres of a particle from a point O after t seconds is given by the equation $s = t^3 - 3t^2 - 9t$. Find the velocity v ($= \frac{ds}{dt}$) in terms of t, and hence find the time at which the particle is stationary (i.e. the velocity is zero).

4. Find $\frac{dy}{dx}$ if:
   (i) $y = (x^2 + 1)(x - 1)$
   (ii) $y = (x - 1)(x + 1)(x - 2)$

5. A curve has equation $y = ax^3 + bx$, where a and b are constants. At the point where $x = 1$, the y–coordinate is 8 and the gradient is 12. Find a and b.

6. Show that the tangent to the curve $y = x^3 + x + 2$ at the point P with x–coordinate 1 passes through the origin, and find the equation of the normal at this point. Given that the normal cuts the x –axis at the point Q, find the area of triangle OPQ.

7. (i) For the graph $y = ax^2 + bx + c$, find the equation of the tangent when $x = p$.
   (ii) Find the equation of the tangent from (i) above, in the case that $b = 0$.
   (iii) Explain by reference to the graph why the answer to (ii) is unchanged for all values of a if $p = 0$.

8. (i) Show that the graphs
   $$y = \tfrac{1}{3}x^3 + 2x + 1 \quad \text{(A)}$$
   $$y = x^2 - \tfrac{1}{2}x + 1 \quad \text{(B)}$$
   cross at the point P with coordinates (0, 1).
   (ii) Find the gradients of the two curves at P.
   (iii) What can you deduce about the two curves from your results in (ii) above?
   (iv) Show that for any value of a, the curve $y = ax^2 - \tfrac{1}{2}x + 1$ crosses the curve (A) above at a constant angle.