Section 1: Points and straight lines

Exercise level 3 (Extension)

1. A triangle has vertices E(2, 5), F(4, 1) and G(-2, -3).
   (i) Find the midpoint of each side and hence find the equations of the three medians.
   (Medians are the lines from the midpoint of each side to the opposite vertex).
   (ii) Show that the point $\left(\frac{4}{3}, 1\right)$ lies on each median.

2. The sides of a triangle are formed by parts of the lines $y + 3x = 11$, $3y = x + 3$ and $7y + x = 37$.
   (i) Find the coordinates of the vertices of the triangle.
   (ii) Show that the triangle is right-angled.
   (iii) Work out the area of the triangle.

3. ABCD is a parallelogram. The equation of AB is $y = 4x - 3$ and the equation of BC is $y = 2x + 1$.
   (i) Find the coordinates of B.
   (ii) The coordinates of A are (3, 9). Find the equation of AD.
   (iii) The coordinates of C are (7, 15). Find the equation of CD.
   (iv) Find the coordinates of D.

4. The perpendicular bisector of AB, where A is (4, 2) and B is (10, 12), crosses the axes at points P and Q. Find the area of triangle OPQ.

5. Point A is (3, 1) and B is (8, 4). A line passes through B perpendicular to AB, and meets the axes at points P and Q. A second line through A perpendicular to AB meets the axes at R and S. Find the area of PQRS. What shape is it?

6. Point A is (5, 2), B is (1, 5), and C is (6, 6). Point D lies on AB, with CD perpendicular to AB. Find the coordinates of D.

7. Point A is (4, 5), B is (2, 1), C is (7, 1), and D is (-1, 5).
   (i) Find the midpoint of AB and CD.
   (ii) Find the gradients of AB and CD.
   (iii) What shape is the figure ACBD?
   (iv) Find the area of figure ACBD.