Section 2: Circles

Exercise level 3 (Extension)

1. Find k so that point P $\left(3, \sqrt{27}\right)$ lies on the circle $x^2 + y^2 = k^2$. If P, Q, and R lie on the circle, and triangle PQR is equilateral, write down the coordinates of the two vertices Q and R.

2. (i) P is point (2, 1) and Q is (10, 5). Find the midpoint M of PQ, and hence write down the equation of the circle with PQ as diameter.
   (ii) Line L$_1$ has equation $y = 3x - 15$. Find the points U, V where line L$_1$ intersects the circle. What is the angle PUQ?
   (iii) Line L$_2$ has equation $y + 2x = 5$. Point R lies on line L$_2$. Find angle RPQ.

3. A set of circles all pass through the points P (1, -3) and Q (5, 7). Show that all their centres lie on a straight line, and find its equation.

4. A gardener is planning an exhibition garden based on a design made up of circles and straight lines. She decides to create a plan, using coordinate geometry, where each unit on her graph represents a distance of 1 metre.
   (i) Write down the equation of a circle centre C (5, 0), with radius 5.
   (ii) On her plan, she draws two straight paths from point P (20, 0) to points Q and R on the circle. Point Q has coordinates (a, b). If she draws PQ so that CQ and PQ are at right angles, what is the length of the path PQ?
   (iii) Find the gradients of the lines CQ and QP in terms of a and b, and hence find the position of Q, and then R.
   (iv) Write down the shape of PQCR, and find its area.