Edexcel AS Mathematics The binomial expansion

Section 1: Finding binomial expansions

Exercise level 3 (Extension)

1. (i) Expand $\left(1+\sqrt{3}\right)^4$

   (ii) Hence write $\left(1-\sqrt{3}\right)^4$ in the form $p+q\sqrt{3}$ where p and q are rational numbers.

2. A process in the aeronautical industry produces cubes, filled with a composite material. If the process is working accurately, the cubes produced are of side length x, and they are then filled with a composite material with density ρ, both measured in suitable units.
   (i) Write down a formula for the mass M of each cube, if the process is working properly.
   (ii) In fact the machine used in the process has small errors in both the length and density, so that the cube dimension is $x + \alpha$, and the density $\rho + \beta$. Write out the binomial expansion of your formula for the mass M, ignoring any terms which involve powers or products of the error terms α and β.
   (iii) It is intended that the production should make cubes with x =3, and ρ = 10, but the errors are measured as α = 0.01 and β = 0.02.
   Use your expansion in (ii) to find an approximation for the extra mass of each cube. (You could check your answer by direct calculation.)

3. Professor Moriarty, the Victorian arch-villain and enemy of Sherlock Holmes, was known to have "written a treatise on the binomial theorem". In one part of his treatise, the professor was investigating the expansion of

$$f(n)=\left(1+\frac{1}{n}\right)^n, \quad n\in\mathbb{Z}^+ \text{ (positive integers)}$$

   (i) Write out fully the expansions of $\left(1+\frac{1}{2}\right)^2$, $\left(1+\frac{1}{3}\right)^3$ and $\left(1+\frac{1}{4}\right)^4$.

   (ii) Write out an approximation for $\left(1+\frac{1}{n}\right)^n$ giving the first 5 terms.

   (iii) The professor extended this experiment by finding approximations for values of n = 10, 100, 1000 (we don't know how he did it!), and went on to suggest a limit for the sequence of numbers he found.
   Use a calculator (or, if available, a spreadsheet) to repeat the experiment and find values for n = 10, 100, 1000., and suggest what your approximation in (ii) leads to as n gets bigger.
   (iv) Experiment with your calculator or spreadsheet using very large numbers to find a more accurate approximation to the limit. (You may find the number you achieve is familiar, and found elsewhere on your calculator.)