Section 3: Matrices and simultaneous equations

Exercise level 2

1. Solve the following simultaneous equations by matrix methods.
    (i) $x - y = 5$
    $3x + 2y = 5$

    (ii) $4x + 6y = 8$
    $2x + 3y = 4$

2. Use a matrix method to solve the equations $x + 3y + z = 4$
    $$2x - y - 4z = 3$$
    $$4x + 5y - 3z = 9$$

3. The matrix $\mathbf{A} = \begin{pmatrix} k & 4 & 0 \\ 0 & k & 4 \\ 2 & 3 & 1 \end{pmatrix}$.

    (i)   State whether $\mathbf{A}$ is singular or non-singular when $k = 2$, justifying your answer.
    (ii)   Determine whether the simultaneous equations below have any solutions
    when $k = 4$.
    $$kx + 4y \quad = 6$$
    $$ky + 4z = 8$$
    $$2x + 3y + z = 1$$

4. (i) Find $\mathbf{AB}$, where $\mathbf{A} = \begin{pmatrix} 2 & -1 & 1 \\ 0 & 3 & 1 \\ 1 & 1 & a \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 3a-1 & a+1 & -4 \\ 1 & 2a-1 & -2 \\ -3 & -3 & 6 \end{pmatrix}$

    Hence write down the inverse matrix $\mathbf{A}^{-1}$, stating a necessary condition on a
    for this inverse to exist.

    (ii) Using this result solve the equations
    $$2x - y + z = 1$$
    $$3y + z = 1$$
    $$x + y + az = 2$$

5. For each of the following values of k, state whether the equations
    $$2x + y + 3z = 5$$
    $$x - 2y + (k+1)z = 2$$
    $$kx + 4y + 2z = 8$$
    have a unique solution, no solutions or infinitely many solutions.
    Give the solutions where they exist, and interpret each situation geometrically.
    (i) $k = -2$
    (ii) $k = 2$
    (iii) $k = 3$

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6. (i) For what value of k is the matrix $\mathbf{A} = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 1 & 1 \\ -3 & 2 & k \end{pmatrix}$ singular?

   (ii) Find $\mathbf{A}^{-1}$ when $k = 3$.

   (iii) If $k = 3.5$ and $\mathbf{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ m \end{pmatrix}$, give the value of m that means that there is a

   solution, and interpret this geometrically.

7. You are given
$$\begin{pmatrix} 3 & -1 & 1 \\ 2 & 1 & -6 \\ 5 & -3 & k \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \\ r \end{pmatrix}$$
There are three possibilities:
   A   there is a unique solution
   B   there are an infinite number of solutions
   C   there are no solutions
What are the conditions on k and r for these possibilities? Interpret your answers geometrically.

8. You are given the equations
   $$x + z = 1$$
   $$2y + 3z = 1$$
   $$x + 7y = 1$$
Find the unique solution to this set of equations
   (i) by writing the equations as a matrix equation and finding the inverse matrix
   (ii) by eliminating x and then z to find y, then substituting to find z and x.
Check that your answers agree.