Section 2: Matrices and transformations

Exercise level 1

1. Draw the triangle OAB with vertices O (0,0), A (1, 3) and B (0, 3).
   Draw the image O′A′B′ of the triangle under the transformation represented by the matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and describe the effect of the transformation.

2. Triangle OAB has vertices O (0, 0), A (2, 1) and B (2, 0).
   For each of the matrices below:
   (a) Draw a diagram to show the effect of the transformation on the triangle OAB, giving the coordinates of the image of A and image of B
   (b) Give a full description of the transformation.
   (i) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ (ii) $\begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$

3. A square has vertices at (0, 0) (1, 1) (0, 2) (-1, 1).
   (i) Write down a matrix **S** that describes this information.
   (ii) Find the image of the square under the transformation $\begin{pmatrix} 4 & 3 \\ -3 & -2 \end{pmatrix}$ by matrix multiplication.
   (iii) Draw both the object and the image on the same diagram.

4. Using matrix multiplication find and draw the image of the square O(0, 0) A(1, 0) B(1, 1) C(0, 1) under the transformation matrix $\begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$.
   Describe the transformation.

5. The transformation represented by $\mathbf{M} = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$ is equivalent to a transformation P, followed by a transformation Q. Give geometrical descriptions of possible transformations P and Q and state the matrices that represent them. Comment on the order in which the transformations are performed.