Section 2: The inverse of a 3×3 matrix

Exercise level 2

1. $\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & x \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 3 & 2 & 1 \\ 4 & 3 & 2 \\ 5 & 4 & x \end{pmatrix}$

   (i) Find $|\mathbf{A}|$ and $|\mathbf{B}|$.
   (ii) For what values of x is $|\mathbf{A}| + |\mathbf{B}| = 0$?
   (iii) For what values of x is $|\mathbf{A}| - |\mathbf{B}| = 0$?

2. The point P is transformed by the matrix $\begin{pmatrix} 3 & -1 & 5 \\ 2 & 0 & -4 \\ 1 & -2 & 0 \end{pmatrix}$ to the image point (2, 10, -1). Find the coordinates of P.

3. Prove that $(\mathbf{ABC})^{-1} = \mathbf{C}^{-1}\mathbf{B}^{-1}\mathbf{A}^{-1}$.

4. **A** and **B** are 3×3 matrices that could be either singular or non-singular. Consider the product **AB**. Is it possible to put 'singular' or 'non-singular' accurately into each box in the grid below?

×	**A** singular	**A** non-singular
B singular		
B non-singular		

5. (i) Without using a calculator, find the inverse of $\mathbf{P} = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -4 \\ 0 & 2 & -19 \end{pmatrix}$ and the inverse of $\mathbf{Q} = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{4} & 0 \end{pmatrix}$.

   (ii) Find **PQ** and $(\mathbf{PQ})^{-1}$, and verify $(\mathbf{PQ})^{-1} = \mathbf{Q}^{-1}\mathbf{P}^{-1}$.

