Section 2: The inverse of a 3×3 matrix

Exercise level 1

1. Without using a calculator, find the determinant of each of these 3 × 3 matrices.

(i) $\begin{pmatrix} 1 & 0 & 2 \\ -3 & 1 & 4 \\ -1 & -3 & -2 \end{pmatrix}$ (ii) $\begin{pmatrix} 2 & 3 & -5 \\ 1 & -2 & -4 \\ 0 & 3 & 0 \end{pmatrix}$ (iii) $\begin{pmatrix} -3 & 1 & 6 \\ -2 & 0 & k \\ 1 & -1 & 4 \end{pmatrix}$

2. $\mathbf{P} = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 0 \\ 3 & 0 & 0 \end{pmatrix}$, $\mathbf{Q} = \begin{pmatrix} 2 & 2 & 2 \\ 3 & 3 & 0 \\ 4 & 0 & 0 \end{pmatrix}$

Without using a calculator, find |**P**|, |**Q**|, **PQ**, **QP**, |**PQ**| and |**QP**|, and hence show that

det(**PQ**) = det(**QP**) = det(**P**) × det(**Q**).

3. A solid shape has volume 5 cm³.

(i) The shape is transformed under the matrix $\mathbf{M} = \begin{pmatrix} 3 & 1 & -2 \\ 2 & 0 & 4 \\ -1 & 0 & -3 \end{pmatrix}$.

What is the volume of the image?

(ii) The original shape is transformed under the matrix $\mathbf{M}^n$. The image has volume 320 cm³. What is the value of n?

4. The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 3 & 0 & 1 \\ 2 & 1 & -2 \\ 4 & 1 & 0 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 2 & -3 \\ -2 & 1 & 1 \\ 3 & -1 & 0 \end{pmatrix}$.

Without using a calculator, find:

(i) $\mathbf{A}^{-1}$ (ii) $\mathbf{B}^{-1}$ (iii) $(\mathbf{AB})^{-1}$ (iv) $(\mathbf{BA})^{-1}$

5. (i) For what value of k is the matrix $\mathbf{M} = \begin{pmatrix} 1 & -3 & 2 \\ 0 & 2 & -2 \\ -1 & 3 & k \end{pmatrix}$ singular?

(ii) If k does not take this value, find $\mathbf{M}^{-1}$ in terms of k.

6. A _____ together with its _____ gives the _____ . The _____ of the _____ matrix gives the _____ matrix. The _____ matrix divided by the _____ gives the _____ matrix.

Fit the following words into the gaps above so that it makes sense and is true.

COFACTOR	ADJUGATE	SIGN
COFACTOR	ADJUGATE	DETERMINANT
MINOR	INVERSE	TRANSPOSE