Section 3: Invariance

Exercise level 2

1. Find any lines of invariant points and invariant lines of the matrix $\begin{pmatrix} 4 & 3 \\ -3 & -2 \end{pmatrix}$.

2. The matrix $\mathbf{M} = \begin{pmatrix} \dfrac{\sqrt{3}}{2} & \dfrac{1}{2} \\ \dfrac{1}{2} & -\dfrac{\sqrt{3}}{2} \end{pmatrix}$

   (i) Calculate $\mathbf{M}^2$.
   (ii) Find the equation of the line of invariant points for this transformation.
   (iii) Describe fully the transformation represented by $\mathbf{M}$ and explain how your answer to part (i) relates to this.
   (iv) Deduce the equations of the other invariant lines.

3. The matrix $\mathbf{S} = \begin{pmatrix} -1 & 1 \\ -4 & 3 \end{pmatrix}$ represents a shear.

   (i) Find the line of invariant points of the shear.
   (ii) Find the invariant lines of the shear.