Section 3: Invariance

Exercise level 1

1. (i)   Show that (1, 3) is an invariant point of the transformation represented by the
          matrix $\begin{pmatrix} -2 & 1 \\ -3 & 2 \end{pmatrix}$.
    (ii) Show that all points of the form $(k, 3k)$ are invariant points of the
          transformation.
    (iii) Hence write down the equation of the line of invariant points.

2. Find any invariant points of the transformations given by
    (i) $\begin{pmatrix} 4 & 3 \\ -3 & -2 \end{pmatrix}$ (ii) $\begin{pmatrix} 0.5 & -0.5 \\ 0.5 & 1.5 \end{pmatrix}$.

3. M is a reflection of the plane such that the image (x', y') of the point (x, y) is given
    by: $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$.
    (i)   Find a point other than the origin that is invariant under this transformation and
          hence find the equation of the mirror line.
    (ii) Find the image A′ of the point A (1, -2) under this transformation.
    (iii) Draw a diagram showing the mirror line, the line $y = -2x$, the point A and the
          point A′. Explain how you know that the line $y = -2x$ is an invariant line for the
          transformation.
    (iv) Write down the equation of two other invariant lines for the transformation.