Section 1: Determinants and inverses

Exercise level 2

1. If $\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 6 & 1 \\ 11 & 3 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} -4 & 3 \\ -5 & 2 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} 4 & 7 \\ -2 & 7 \end{pmatrix}$, find matrices **X**, **Y** and **Z** such that **AX** = **B**, **BY** = **C**, **CZ** = **D**.

2. The matrix $\begin{pmatrix} a-3 & -2 \\ a & 2a-1 \end{pmatrix}$ is singular. Find the possible values of a.

3. The matrix $\mathbf{M} = \begin{pmatrix} 2 & 2 \\ -3 & 1 \end{pmatrix}$ represents a transformation of the (x, y) plane.

   A trapezium T with area 4 square units is transformed by **M** into trapezium T'.
   (i) Find the area of the new trapezium T'
   (ii) Find the matrix which transforms T' into T.

4. The plane is transformed by means of the matrix $\mathbf{M} = \begin{pmatrix} 3 & 6 \\ 1 & 2 \end{pmatrix}$.

   Show that det **M** = 0 and **M** maps all points of the xy plane onto a straight line. Give the equation of the line.

5. Transformations S and R are represented by matrices **M** and **N** respectively, where $\mathbf{M} = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$ and $\mathbf{N} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$.
   (i) Describe the transformations represented by S and R.
   (ii) Find $\mathbf{M}^{-1}$ and $\mathbf{N}^{-1}$.
   (iii) Find **MN** and $(\mathbf{MN})^{-1}$. Verify that $(\mathbf{MN})^{-1} = \mathbf{N}^{-1}\mathbf{M}^{-1}$
   (iv) Explain the result $(\mathbf{MN})^{-1} = \mathbf{N}^{-1}\mathbf{M}^{-1}$ in terms of the transformations S and R.

6. Show that the matrix $\begin{pmatrix} 3 & 6 \\ -2 & -4 \end{pmatrix}$ maps all points of the x-y plane onto a straight line and find the equation of that line.

7. Matrix **A** represents a transformation T where $\mathbf{A} = \begin{pmatrix} 2 & -3 \\ 4 & -4 \end{pmatrix}$.
   (i) Find the inverse of **A.**
   (ii) Find the coordinates of the point that is mapped to (9, 16) under transformation T.
   (iii) Find $\mathbf{A}^2$.
   (iv) Show that $\mathbf{A}^3 = \begin{pmatrix} d & 0 \\ 0 & d \end{pmatrix}$ stating the value of d.
   (v) Give a geometrical description of the matrix $\mathbf{A}^3$.

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8. The matrix $\mathbf{A} = \begin{pmatrix} 2 & -3 \\ -2 & 4 \end{pmatrix}$

   (i) Find $\mathbf{A}^{-1}$.

   (ii) Given that $\mathbf{A} = \mathbf{BC}$, where $\mathbf{B} = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$, find $\mathbf{C}^{-1}$.

9. Find the value of x given that $\mathbf{A}^2 = \mathbf{A}^{-1}$ and

   $\mathbf{A} = \begin{pmatrix} 1 & x & 1 \\ -1 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$.