Section 1: Determinants and inverses

Exercise level 1

1. Find the inverses of the following matrices, if they exist.

   (i) $\mathbf{A} = \begin{pmatrix} 3 & -1 \\ 2 & 1 \end{pmatrix}$ (ii) $\mathbf{B} = \begin{pmatrix} 4 & -2 \\ -2 & 1 \end{pmatrix}$

2. Use the matrix facility on your calculator to find the inverses of the following matrices. Check your answers by multiplying.

   (i) $\mathbf{P} = \begin{pmatrix} 1 & 3 & -2 \\ 0 & 2 & 1 \\ 5 & -1 & 2 \end{pmatrix}$ (ii) $\mathbf{Q} = \begin{pmatrix} 2 & 3 & 1 \\ -1 & 1 & 2 \\ 1 & 2 & 0 \end{pmatrix}$

3. If $\mathbf{B} = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$ and $\mathbf{AB} = \mathbf{I}$ find $\mathbf{A}$.

4. Find $\mathbf{B}$ if $\mathbf{A} = \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$ and $\mathbf{AB} = \begin{pmatrix} 4 & -2 \\ 0 & 7 \end{pmatrix}$.

5. Find $\mathbf{Y}$ if $\mathbf{Y}\begin{pmatrix} -2 & 0 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

6. If the matrix $\begin{pmatrix} 1 & -2 \\ x & 4 \end{pmatrix}$ has no inverse, find x.

7. $\mathbf{P} = \begin{pmatrix} 2 & 3 & 2 \\ 4 & 6 & 5 \\ 5 & 7 & 6 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 1 & -4 & 3 \\ 1 & 2 & -2 \\ -2 & 1 & 0 \end{pmatrix}$.

   Find $\mathbf{PQ}$ and $\mathbf{QP}$. What can you deduce from your answers?

8. The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} 2 & 4 \\ 0 & 2 \end{pmatrix}$

   (i) Draw a diagram showing the unit square and its image under the transformation represented by $\mathbf{M}$.

   (ii) Calculate the determinant of $\mathbf{M}$ and explain how this value relates to the transformation represented by $\mathbf{M}$.