Section 1: Modulus and argument

Exercise level 2

1. Given that $z_1 = 12 + 5\text{i}$ and $z_2 = -3 + 4\text{i}$, verify that $|z_1 + z_2| \le |z_1| + |z_2|$.
   Explain geometrically using an Argand diagram why $|z_1 + z_2| \le |z_1| + |z_2|$ is always true.

2. Given that $z_1 = 12 + 5\text{i}$ and $z_2 = 3 - 4\text{i}$ verify that $|z_1 - z_2| \ge |z_1| - |z_2|$. With reference to an Argand diagram give a geometric explanation of this result.

3. Write each of the following in modulus-argument form.
   (i) $-2\sqrt{3} - 2\text{i}$ (ii) $\dfrac{10}{\sqrt{3} - \text{i}}$

4. Given that $z = 1 + 2\text{i}$, write in modulus-argument form the complex numbers
   (i) z (ii) z^* (iii) $\dfrac{1}{z}$ (iv) $\dfrac{1}{z^*}$
   What do you notice?

5. Given that $w = 10\text{i}$ and $z = 1 + \sqrt{3}\text{i}$
   (i) write each of w and z in the form $r(\cos\theta + \text{i}\sin\theta)$, where $r > 0$ and $-\pi < \theta \le \pi$.
   (ii) find wz, and $\dfrac{w}{z}$ in the form $r(\cos\theta + \text{i}\sin\theta)$, where $r > 0$ and $-\pi < \theta \le \pi$.

6. Given that $z = \cos\theta + \text{i}\sin\theta$ find arg $(z + 1)$. (Hint: draw an Argand diagram and use the double angle formulae $\sin 2\theta = 2\sin\theta\cos\theta$ and $\cos 2\theta = 2\cos^2\theta - 1$.)

7. The complex numbers α and β are given by $\alpha = 1 - \sqrt{3}\text{i}$ and $\beta = -2 + 2\text{i}$.
   (i) Find $|\alpha|$, $|\beta|$, $\arg\alpha$ and $\arg\beta$.
   (ii) Show the points A and B, representing α and β respectively, on an Argand diagram.
   (iii)Find $\dfrac{\beta}{\alpha}$ in the form $r(\cos\theta + \text{i}\sin\theta)$, where $r > 0$ and $-\pi < \theta \le \pi$.
   (iv)Hence describe fully the transformation which maps the line OA to the line OB.

8. Let $z_1 = -1 + \text{i}$ and $z_2 = \sqrt{3} + \text{i}$.
   (i) Write z_1 and z_2 in polar form and hence write $\dfrac{z_1}{z_2}$ in polar form.
   (ii) Write $\dfrac{-1+\text{i}}{\sqrt{3}+\text{i}}$ in the form $a + b\text{i}$.
   (iii) Hence find the exact values of $\cos\left(\dfrac{7\pi}{12}\right)$ and $\sin\left(\dfrac{7\pi}{12}\right)$.