Section 2: Loci in the complex plane

Notes and Examples

These notes contain subsections on:

- Circles
- Half-lines
- Perpendicular bisectors

There are several different types of loci which you need to be familiar with.

Circles

A set of points of the form $|z - z_1| = r$ represents a circle.

Read $|z – (a + b\text{i})|$ as 'the distance from z to the point $a + b\text{i}$
[note $|z – a + b\text{i}|$ can be written $|z – (a – b\text{i})|$, which is the distance from z to the point $(a – b\text{i})$].

All sets of points given by $|z – (a + b\text{i})| = r$ can be represented by a circle, centre $a + b\text{i}$, radius r..

$|z – (a + b\text{i})| \leq r$ represents the circle and its interior.

$|z – (a + b\text{i})| < r$ represents the interior of the circle (but not the circle itself). In this case you should draw the circle as a dotted line, to show that it is not included in the set of points.

$|z – (a + b\text{i})| \geq r$ represents the circle and its exterior.

$|z – (a + b\text{i})| > r$ represents the exterior of the circle (but not the circle itself). Again, you should draw the circle as a dotted line, to show that it is not included in the set of points.

Half-lines

A set of points of the form $\arg(z – z_1) = \theta$ represents a half-line

The point z satisfies this locus when the line joining z_1 to z has argument θ.

All sets of points of the form $\arg(z-(a+b\text{i}))=\theta$ consist of a half-line from the point $a + b\text{i}$ in the direction θ.

Sets of points of the form $\theta_1 \leq \arg(z-(a+b\text{i})) \leq \theta_2$ consist of the two half-lines $\arg(z-(a+b\text{i}))=\theta_1$ and $\arg(z-(a+b\text{i}))=\theta_2$ and the region between them .

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Be careful with sets of points of the form $\arg(z-(a+bi)) \leq \theta$ or $\arg(z-(a+bi)) \geq \theta$. Make sure that you know where the region starts and ends. Remember that the possible values of $\arg z$ are given by $-\pi < \arg z \leq \pi$, and that any line which is not included in the set of points should be shown as dotted (see Example 1 below).

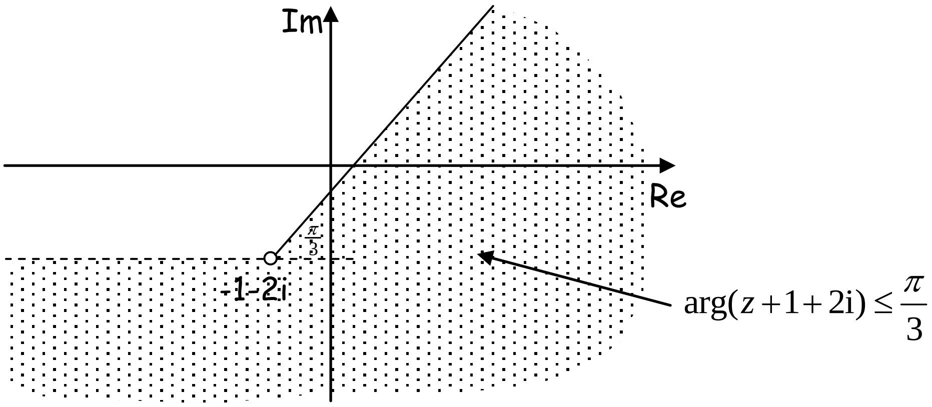
Example 1
Draw Argand diagrams to show each of the following sets of points.

(i) $\arg(z+1+2\text{i}) \leq \dfrac{\pi}{3}$

(ii) $\arg(z-2-\text{i}) > \dfrac{3\pi}{4}$

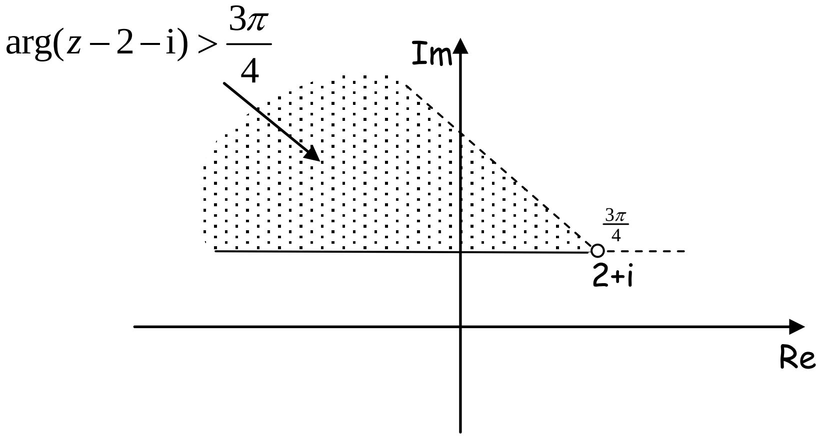
Solution

(i) $\arg(z+1+2\text{i}) \leq \dfrac{\pi}{3}$ means that $\arg(z+1+2\text{i})$ can take any value between $-\pi$ and $\dfrac{\pi}{3}$, including $\dfrac{\pi}{3}$ but not $-\pi$. The half-line from $-1 - 2\text{i}$ in the direction $-\pi$ is therefore shown dotted, and the half-line from $-1 - 2\text{i}$ in the direction $\dfrac{\pi}{3}$ is shown as solid. The point $-1 - 2\text{i}$ is not included in the region, since $\arg(z+1+2\text{i})$ is not defined where $z = -1 - 2\text{i}$, so this point is shown by an open circle.



(ii) $\arg(z-2-\text{i}) > \dfrac{3\pi}{4}$ means that $\arg(z-2-\text{i})$ can take any value between $\dfrac{3\pi}{4}$ and π, including π but not $\dfrac{3\pi}{4}$ since the inequality involves $>$ rather than $\geq$.

The half-line from $2 + \text{i}$ in the direction $\dfrac{3\pi}{4}$ is therefore shown dotted, and the half-line from $2 + \text{i}$ in the direction π is shown solid. The point $2 + \text{i}$ is not included in the region, since $\arg(z-2-\text{i})$ is not defined where $z = 2 + \text{i}$, so this point is shown by an open circle.



Perpendicular bisectors

A locus of the form $|z - z_1| = |z - z_2|$ represents a straight line

The equation $|z-(a+b\mathrm{i})| = |z-(c+d\mathrm{i})|$ means that z is the same distance from the points representing the complex numbers $a + b\mathrm{i}$ and $c + d\mathrm{i}$. The set of points for which this is true is the perpendicular bisector of a line connecting the points $a + b\mathrm{i}$ and $c + d\mathrm{i}$. The locus of z for $|z - z_1| = |z - z_2|$ is therefore the perpendicular bisector of the line joining z_1 to z_2 .

$|z-(a+b\mathrm{i})| \le |z-(c+d\mathrm{i})|$ means that z is nearer to $a + b\mathrm{i}$ than to $c + d\mathrm{i}$, so this represents the side containing the point $a + b\mathrm{i}$ of the perpendicular bisector of the line joining $a + b\mathrm{i}$ to $c + d\mathrm{i}$. As with the circles, in the case of ≤ or ≥ the perpendicular bisector itself is included, so is shown in as a solid line, but for < or > the line must be shown as dotted as it is not included.

Be careful not to mix up a perpendicular bisector locus with a circle locus!