Section 2: The Argand diagram

Notes and Examples

These notes contain subsections on

- Representing complex numbers geometrically
- Addition and subtraction in the Argand diagram

Representing complex numbers geometrically

The Notes and Examples for Section 1 looked at the relationships between numbers as represented in a Venn diagram, with some sets of numbers being a subset of another set: e.g. the integers are a subset of the rational numbers. You have seen that all the types of number that you have met so far can be considered to be a subset of a larger set of numbers: the complex numbers. This can be represented on the Venn diagram by a larger set encircling the set representing the real numbers.

Another way to represent numbers is on a number line. You have probably used number lines from a very early stage in your mathematical development. Even irrational numbers can be placed on a number line: for example, $\sqrt{2}$ can be expressed to as many decimal places as you like.

However, if you want to place a complex number on the number line, you have a problem. Is 1 + i larger or smaller than 1? Clearly this kind of question just does not make sense.

The Argand diagram provides a way of representing complex numbers geometrically, in the same way that a number line can represent the real numbers.

Example 1
The complex numbers z and w are given by

$z = 3 - 2\text{i}$

$w = -1 + 4\text{i}$

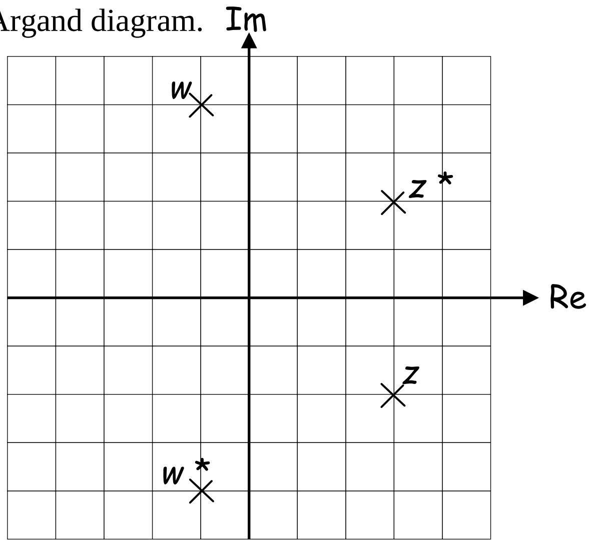
Plot the points z, w, z^* and w^* on an Argand diagram.

Solution
$z = 3 - 2\text{i}$ is represented by the point (3, -2)

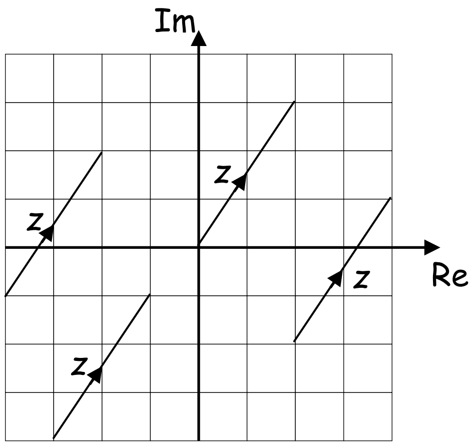
$z^* = 3 + 2\text{i}$ is represented by the point (3, 2)

$w = -1 + 4\text{i}$ is represented by the point (-1, 4)

$w^* = -1 - 4\text{i}$ is represented by the point (-1, -4)



As well as thinking of a complex number $z = x + yi$ as a point with coordinates (x, y), you can also think of it as a vector $\begin{pmatrix} x \\ y \end{pmatrix}$. This could be a position vector (a vector from the origin to the point (x, y)) but it can be any vector (sometimes called a directed line segment) parallel to this.

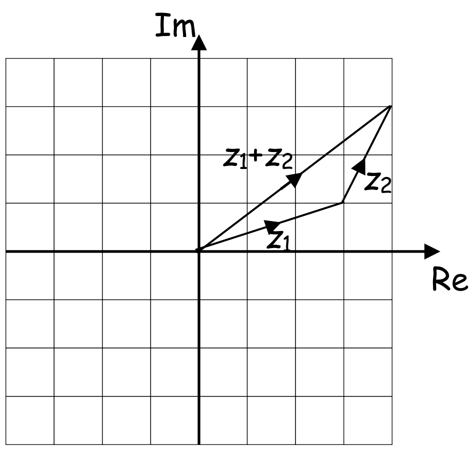


All the vectors on this diagram represent the complex number $z = 2 + 3i$. Notice that it is the vector itself that is labelled z, not the point at the end of it.

Addition and subtraction in the Argand diagram

Addition of two complex numbers

Here $z_1 = 3 + i$ and $z_2 = 1 + 2i$. You can see from the diagram that $z_1 + z_2 = 4 + 3i$, as you would expect from adding z_1 and z_2 together.



Subtraction of two complex numbers

You can think of subtraction in two different ways: either by thinking of $z_1 - z_2$ as adding together the vectors z_1 and $-z_2$ (shown in the diagram on the left) or by going from the point z_2 to the point z_1 (shown in the diagram on the right).

In either case, with $z_1 = 3 + \mathrm{i}$ and $z_2 = 1 + 2\mathrm{i}$, you can see that the vector $z_1 - z_2$ is given by $2 - \mathrm{i}$, the result you would expect from subtracting the complex number z_2 from z_1.