Section 1: Introduction to complex numbers

Exercise level 2

1. $z = -3 + 4\text{i}$ and $w = \dfrac{5 + 2\text{i}}{z}$

   Find w, giving your answer in the form $a + b\text{i}$, where a and b are real.

2. Given that $z = (a + \text{i})^4$ where a is real, find values for a such that
   (i) z is real,
   (ii) z is wholly imaginary.

3. Given that $a + b\text{i}$ is the conjugate of $(a + b\text{i})^2$ find **all** possible pairs of values for a and b.

4. Simplify and write in the form $a + b\text{i}$:
   (i) $\dfrac{1}{3+2\text{i}} + \dfrac{1}{3-2\text{i}}$
   (ii) $3 + \text{i} + \dfrac{4}{3-\text{i}}$
   (iii) $\dfrac{3}{1-\text{i}} - \dfrac{2\text{i}}{2+\text{i}}$

5. Find values for a and b that satisfy each of the following:
   (i) $(a + b\text{i})(2 + \text{i}) = a - 3\text{i}$
   (ii) $(a + \text{i})(4 - b\text{i}) = 3b + 2a\text{i}$

6. By writing $(a + b\text{i})^2 = 3 - 4\text{i}$, find values for a and b and hence find the square roots of $3 - 4\text{i}$.

7. Find the values of p and q given that one root of the equation $z^2 + pz + q = 0$ is:
   (i) $2 - \text{i}$ (ii) $1 - 3\text{i}$
   (iii) 2i (iv) $5 - 3\text{i}$

8. Given that $\dfrac{5}{a + b\text{i}} + \dfrac{2}{1 + 3\text{i}} = 1$, where a and b are real, find the values of a and b.